CHAPTER 5

Introduction to Swap Engineering

1. The Swap Logic

Swaps are the first basic tool that we introduced in Chapter 1. It should be clear by now that swaps are essentially the generalization of what was discussed in Chapters 1, 3 and 4. We start this chapter by providing a general logic for swaps.

It is important to realize that essentially all swaps can be combined under one single logic. Consider *any* asset. Suppose we add to this asset another contract and form a basket. But, suppose we choose this asset so that the market risk, or the volatility associated with it, is exactly zero. Then the volatility (or the risk) of the basket is identical to the volatility (or the risk) of the original asset. Yet, the addition of this "zero" can change other characteristics of the asset and make the whole portfolio much more liquid, practical and useful for hedging, pricing and administrative reasons. This is what happens when we move from original "cash" securities to swaps. We take a security and augment it with a "zero volatility" asset. This is the swap strategy.

1.1. The Equivalent of Zero in Finance

First we would like to develop the equivalent of zero in finance as was done in Chapter 1. Why? Because, in the case of standard algebra, we can add (subtract) zero to a number and its value does not change. Similarly if we have the equivalent of a "zero" as a security, then we could add this security to other securities and this addition would not change the original *risk characteristics* of the original security. But in the mean time, the cash flow characteristics, regulatory requirements, tax exposure and balance sheet exposure of the portfolio may change in a desirable way.

What is a candidate for such a "zero"? Consider the interbank money market loan in Figure 5-1. The loan principal is 100 and is paid at time t_1. Interest and principal is received at t_2. Hence this is a default-free loan to be made in the future. The associated interest rate is the Libor rate L_{t_1} to be observed at time t_1.

We write a forward contract on this loan. According to this 100 is borrowed at t_1 and for this the prevailing interest rate is paid at that time. What is, then, the value of this forward loan contract for all $t \in [t_0, t_1]$?

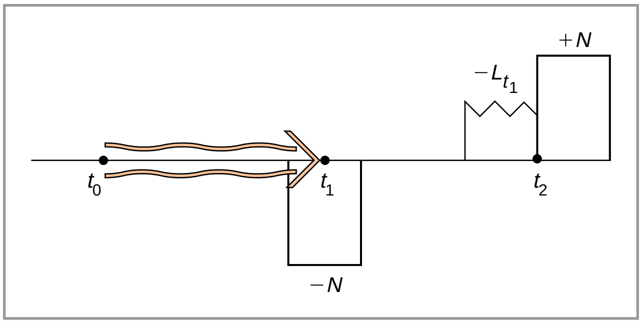


FIGURE 5-1

It turns out that one can, in fact, calculate this value exactly at time t_0 even though the future Libor rate L_{t_1} is not known then. Consider the following argument.

The t_2-cash flows are

$$+100 + 100L_{t_1}\delta \tag{1}$$

Discounting this value to time t_1 we get:

$$\frac{+(1 + L_{t_1}\delta)100}{(1 + L_{t_1}\delta)} = +100 \tag{2}$$

Adding this to the initial 100 that was lent, we see that the total value of the cash flows generated by the forward loan contract is exactly zero for all times t during the interval $[t_0, t_1]$, *no matter what* the market thinks about the future level of L_{t_1}.[1]

Denoting the value of this forward contract by V_t, we can immediately see that:

$$\text{Volatility } (V_t) \equiv 0 \quad \text{For all } t \in [t_0, t_1] \tag{3}$$

Hence adding this contract to any portfolio would not change the risk (volatility) characteristics of that portfolio. This is important and is a special property of such Libor contracts.[2] Thus let V_t denote the value of a security with a sequence of cash flows so that the security has a value equal to zero identically for all $t \in [t_0, t_1]$,

$$V_t = 0 \tag{4}$$

Let S_t be the value of any other security, with

$$0 < \text{Volatility } (S_t) \quad t \in [t_0, t_1] \tag{5}$$

[1] Another way of saying this is to substitute the forward rate F_{t_0} for L_{t_1}. As Δ amount of time passes this forward rate would change to $F_{t_0+\Delta}$. But the value of the loan would not change, because

$$\frac{-(1 + F_{t_0}\delta)100}{(1 + F_{t_0}\delta)} = \frac{-(1 + F_{t_0+\Delta}\delta)100}{(1 + F_{t_0+\Delta}\delta)} = -100$$

[2] For example, if the forward contract specified a forward rate F_{t_0} at time t_0, the value of the contract would not stay the same, since starting from time t_0 as Δ amount of time passes, a forward contract that specifies a F_{t_0} will have the value:

$$\frac{-(1 + F_{t_0}\delta)100}{(1 + F_{t_0+\Delta}\delta)} \neq \frac{-(1 + F_{t_0}\delta)100}{-(1 + F_{t_0}\delta)} = -100$$

This is the case since, normally,

$$F_{t_0} \neq F_{t_0+\Delta}$$

Suppose both assets are *default-free*. Then, because the loan contract has a value identically equal to zero for *all* $t \in [t_0, t_1]$ we can write,

$$S_t + V_t = S_t \tag{6}$$

in the sense that,

$$\text{Volatility } (S_t + V_t) = \text{Volatility } (S_t) \tag{7}$$

Hence the portfolio consisting of an S_t *and* a V_t asset has the identical volatility and correlation characteristics as the original asset S_t. It is in this sense that the asset V_t is equivalent to zero. By adding it to any portfolio we do not change the market risk characteristics of this portfolio.

Still, the addition of V_t may change the original asset in important ways. In fact, with the addition of V_t

1. The asset may move the S_t off-balance sheet. Essentially, nothing is purchased for cash.
2. Registration properties may change. Again no basic security is purchased.[3]
3. Regulatory and tax treatment of the asset may change.
4. No upfront cash will be needed to take the position. This will make the modified asset much more liquid.

We will show these using three important applications of the swap logic. But first some advantages of the swaps. Swaps have the following important advantages among others.

Remark 1: When you buy a U.S. Treasury bond or a stock issued by a U.S. company, you can only do this in the United States. But, when you work with the swap, $S_t + V_t$, you can do it anywhere, since you are *not* buying/selling a cash bond or a "cash" stock. It will consist of only swapping cash.

Remark 2: The swap operation is a natural extension of a market practitioner's daily work. When a trader buys an asset the trader needs to *fund* this trade. "Funding" an asset with a Libor loan amounts to the same scheme as adding V_t to the S_t. In fact, the addition of the zero asset eliminates the initial cash payments.

Remark 3: The new portfolio will have no *default* risk.[4] In fact with a swap, no loan is extended by any party.

Remark 4: Finally the accounting, tax and regulatory treatment of the new basket may be much more advantageous.

1.2. A generalization

We can generalize this notion of "zero." Consider Figure 5-2. This figure adds *vertically* n such deposits, all having the same maturity but starting at different times, $t_i, i = 1, 2, \ldots$. The resulting cash flows can be interpreted in two ways. First, the cash flows can be regarded as

[3] What is purchased is its derivative.

[4] Although there will be a counterparty risk.

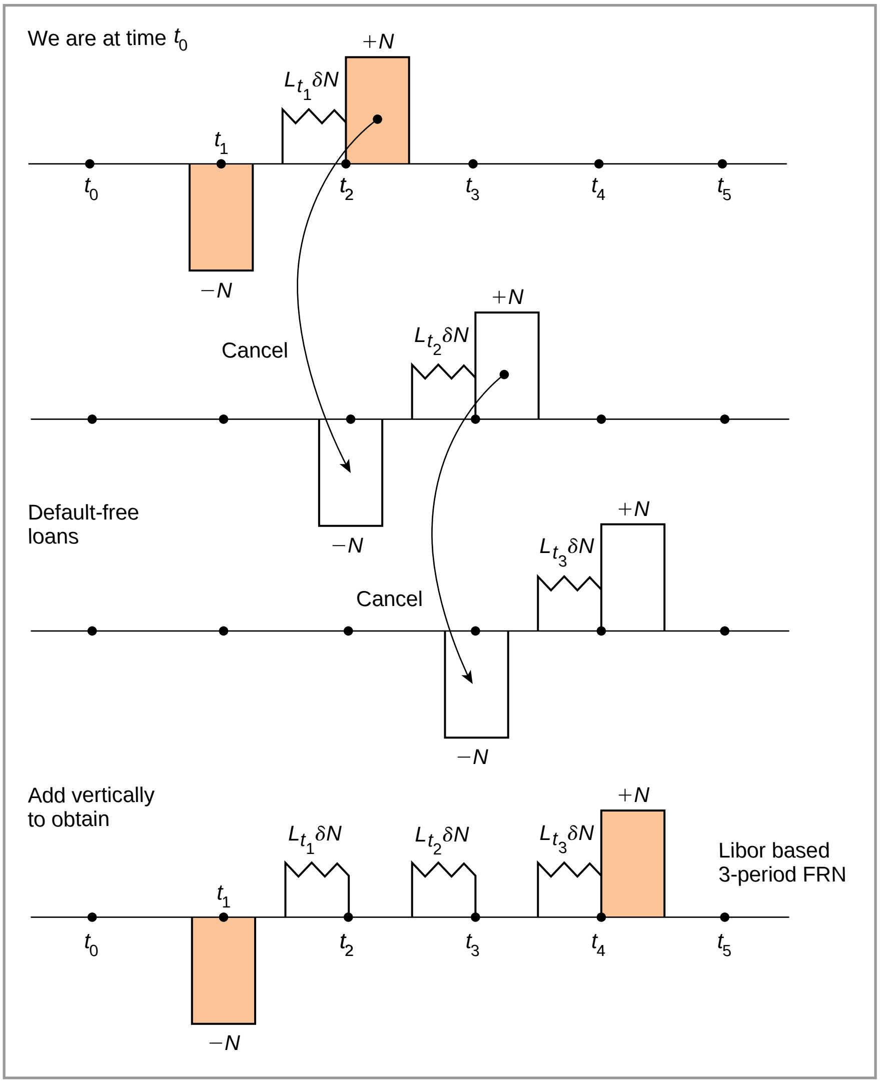


FIGURE 5-2

coming from a Floating Rate Note (FRN) that is purchased at time t_i with maturity at $t_n = T$. The note pays Libor flat. The value of the FRN at time t_i will be given by

$$\begin{aligned} \text{Value}_t[\text{FRN}] &= V_t^1 + V_t^2 + \cdot\cdot\cdot + V_t^n \quad t \in [t_0, t_1] \\ &= 0 \end{aligned} \tag{8}$$

Where the V_t^i is the time t value of the period deposit starting at time t_1.

The second interpretation is that the cash flows shown in Figure 5-2 are those of a *sequence* of money market loans that are rolled over at periods $t_1, t_2, . \ . \ . , t_{n-1}$.

2. Applications

In order to see how powerful such a logic can be, we apply the procedure to different types of assets as was done in Chapter 1. First we consider an equity portfolio and add the zero-volatility asset to it. This way we obtain an *equity swap*. A *commodity swap* can be obtained similarly.

Then we do the same with a *defaultable bond*. The operation will lead to a Credit Default Swap (CDS). The modification of this example will lead to the use of a default-free bond and will result in an Interest Rate Swap.

These swaps lead to some of the most liquid and largest markets in the world. They are all obtained from a single swap logic.

2.1. Equity Swap

Consider a portfolio of stocks whose *fair* market value at time t_0 is denoted by S_{t_0}. Let $t_n = T, t_0 < \cdots < t_n$ where the T is a date that defines the expiration of an equity swap contract. For simplicity think of $t_n - t_0$ as a one-year period. We divide this period into equally spaced intervals of length δ, with $t_1, t_2, t_3, \ldots, t_n = T$ being the settlement dates.

Let $\delta = \frac{1}{4}$ so that the t_i are 3 months apart. During a one-year interval with $n = 4$, the portfolio's value will change by:

$$S_{t_4} - S_{t_0} = [(S_{t_1} - S_{t_0}) + (S_{t_2} - S_{t_1}) + (S_{t_3} - S_{t_2}) + (S_{t_4} - S_{t_3})] \tag{9}$$

This can be rewritten as

$$S_{t_4} - S_{t_0} = \Delta S_{t_0} + \Delta S_{t_1} + \Delta S_{t_2} + \Delta S_{t_3} \tag{10}$$

We consider buying and marking this portfolio to market in the following manner.

1. $N = 100$ is invested at time t_1.
2. At every $t_1, i = 1, 2, 3, 4$ total dividends amounting to d are collected.[5]
3. At the settlement dates we collect (pay) the cash due to the appreciation (depreciation) of the portfolio value.
4. At time $t_n = T$ collect the original USD100 invested.

This is exactly what an equity investor would do. The investor would take the initial investment (principal), buy the stocks, collect dividends and then sell the stocks. The final capital gains or losses will be $S_{t_n} - S_{t_0}$. In our case, this is monetized at each settlement date. The cash flows generated by this process can be seen in Figure 5-3.

Now we follow the swap logic discussed above and add to the stock portfolio the contract V_t which denotes the time t value of the cash flows implied by a forward Libor-deposit. Let g_{t_i} be the percentage decline or increase in portfolio value at each and let the initial investment be denoted by the notional amount N:

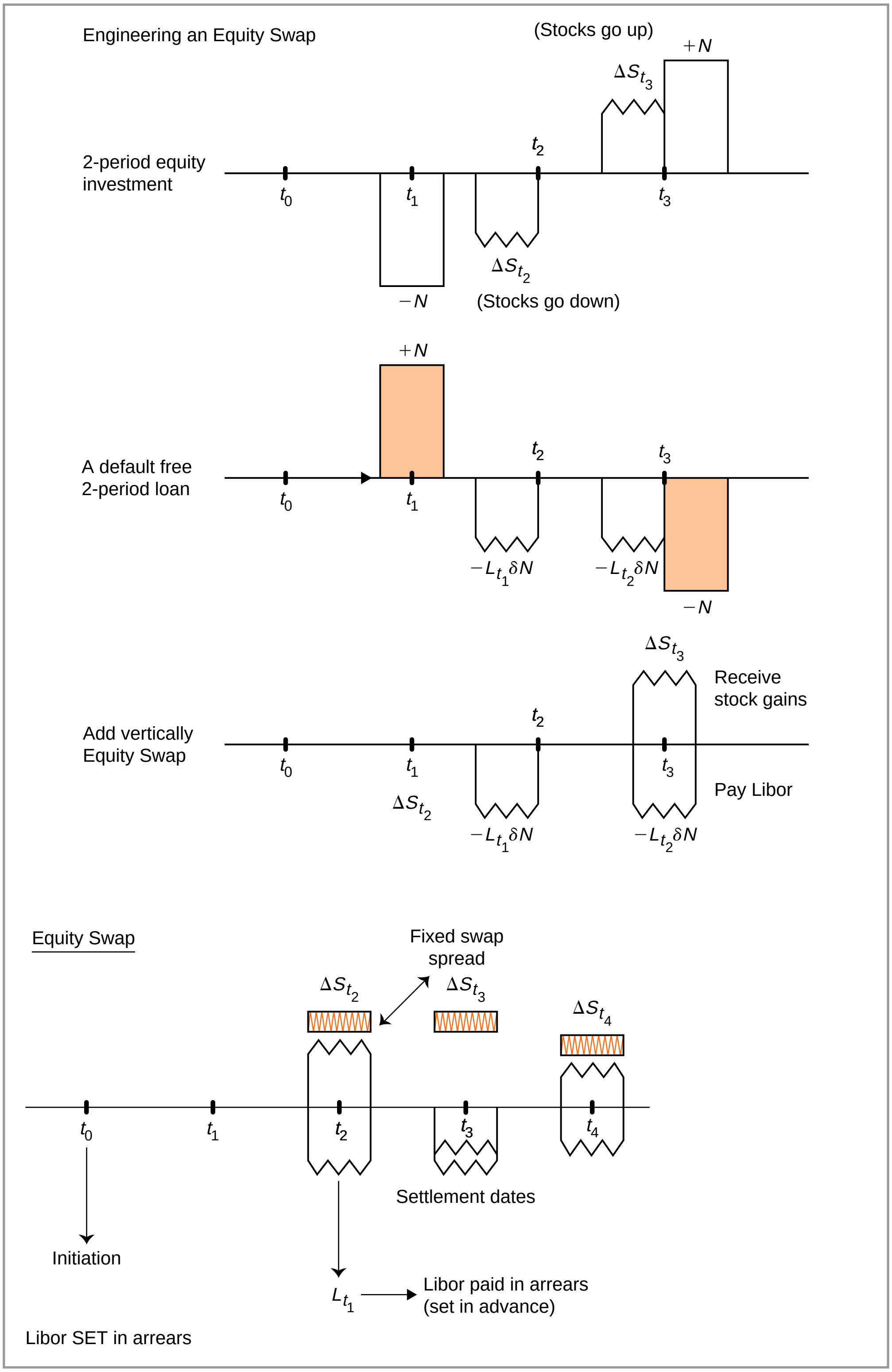
$$S_{t_0} = N \tag{11}$$

Then,

1. The value of the stock portfolio has not changed any time between t_0 and t_1, since the forward FRN has value identically equal to zero at any time $t \in [t, t_0]$.
2. But the initial and final N's cancel.
3. The outcome is an exchange of

$$L_{t_{i-1}} \delta N \tag{12}$$

[5] Note that we are assuming constant and known dividend payments throughout the contract period.



Engineering an Equity Swap
(Stocks go up)
+N
ΔS_{t_3}
2-period equity investment
t_0
t_1
t_2
t_3
ΔS_{t_2}
−N
(Stocks go down)
+N
A default free 2-period loan
t_0
t_1
t_2
t_3
$-L_{t_1}\delta N$
$-L_{t_2}\delta N$
−N
ΔS_{t_3}
Receive stock gains
Add vertically Equity Swap
t_0
t_1
t_2
t_3
ΔS_{t_2}
Pay Libor
$-L_{t_1}\delta N$
$-L_{t_2}\delta N$
Equity Swap
Fixed swap spread
ΔS_{t_2}
ΔS_{t_3}
ΔS_{t_4}
t_0
t_1
t_2
t_3
t_4
Settlement dates
Initiation
L_{t_1}
Libor paid in arrears (set in advance)
Libor SET in arrears

FIGURE 5-3

against

$$(\Delta S_{t_i} + d)\delta N \tag{13}$$

at each t_i.

4. Then we can express the cash flows of an equity swap as the exchange of

$$(L_{t_i} - d_i)\delta N \tag{14}$$

against

$$\Delta S_{t_i}\delta N \tag{15}$$

at each t_i. The d_i being an unknown percentage dividend yield, the market will trade this as a spread. The market maker will quote the "expected value of" d_i and any incremental supply-demand imbalances as the *equity swap spread*.

5. The swap will involve no upfront payment.

This construction proves that the market *expects* the portfolio S_{t_i} to change by $L_{t_{i-1}} - d_{t_i}$ each period, in other words, we have,

$$E_t^{\overline{P}}[\Delta S_{t_i}] = L_{t_{i-1}} - d_{t_i} \tag{16}$$

This result is proved normally by using the fundamental theorem of asset pricing and the implied risk-neutral probability.

2.2. Commodity Swap

Suppose the S_t discussed above represents not a stock, but a commodity. It could be oil for example. Then, the analysis would be identical in engineering a commodity swap.

One could invest $N = 100$ and "buy" Q units of the commodity in question. The price S_t would move over time. One can think of investment paying (receiving) any capital gains (losses) to the investor at regular intervals, $t_0, t_1, \ldots, t_n$. At the maturity of the investment the N is returned to the investor. All this is identical to the case of stocks.

One can put together a *commodity swap* by adding the n-period FRN to this investment. The initial and final payments of the N would cancel and the swap would consist of paying any capital gains and receiving the capital losses and the $Libor + d_t$, where the d_t is the swap spread.

Note that the swap spread may deviate from zero due to any *convenience yield* the commodity may offer, or due to supply demand imbalances during short periods of time. The convenience yield here would be the equivalent of the dividends paid by the stock.

2.3. Cross Currency Swap

Can a commodity swap structure be applied to currencies? The answer is positive. Suppose the "commodity" we buy with the initial $N = 100$ is a foreign currency, and the s_t is the exchange rate. Thus we are buying Q units of the foreign currency at the dollar price of s_t.[6] We have

$$N = Qs_t \tag{17}$$

[6] This means the foreign currency is considered to be the base currency.

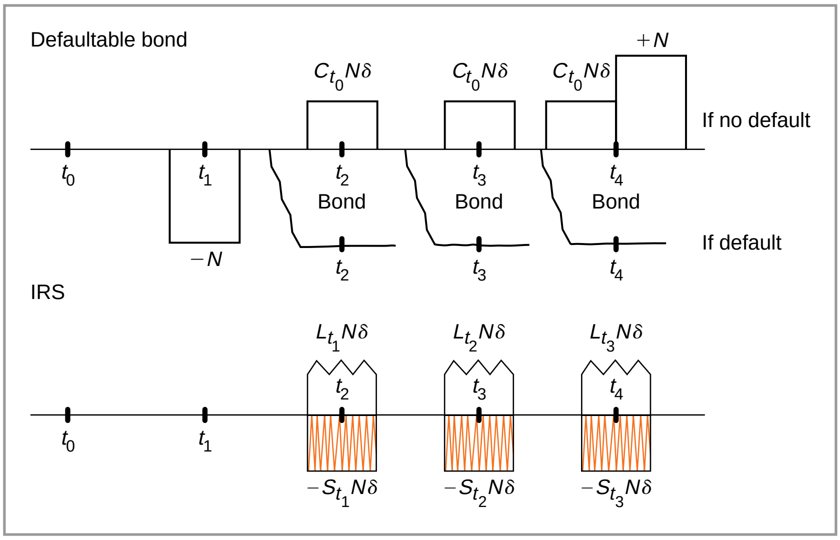


FIGURE 5-4

Then we can put together a swap that pays capital gains on the foreign exchange bought and the interest generated by this foreign exchange (supposedly foreign currency Libor) and receives the capital losses plus Libor.

There is, however, an important special characteristic of the cross-currency swaps. Often, the “notional” amounts are exchanged at initiation and at maturity. See Figure 5-4.

2.4. Engineering a CDS

We can apply the same technique to a *defaultable* bond shown in Figure 5-5a. The bond pays coupon c_{t_0}, has par value N, and matures, without loss of generality, in three years. It carries a default risk as shown in the cash flow diagram. If the bond defaults the bond holder will have a *defaulted* bond in his hand. Otherwise the bond holder receives the coupons and the principal. Note that there are only three default possibilities at the three settlement dates, t_1, t_2, t_3.

The market practitioner buys the bond with a floating rate loan that is rolled at every settlement date. This situation is shown in Figure 5-5a. Clearly it is equivalent to adding the “zero” to the defaultable bond. Adding vertically, we get the cash flow diagram in Figure 5-5b. To convert this into a default swap one final operation is needed.

The libor payments are equivalent to three fixed payments at the going swap rate s_{t_0} as shown in Figure 5-5c. Adding this swap to the third diagram in Figure 5-5c we obtain the cash flows in Figure 5-6. This is a credit default swap. Essentially it is a contract,

1. Where the protection seller (defaultable bond holder) receives the spread

$$Sp_{t_0} = cds_{t_0} = c_{t_0} - s_{t_0} \tag{18}$$

   at each settlement date t_1, t_2, t_3,
2. But makes a payment of $(1 + s_{t_0}\delta)N$ as soon as default occurs.
3. Against this compensation for default, the protection seller receives the physical delivery of the defaulted bonds of face value N.

Now we move to interest rate swaps.

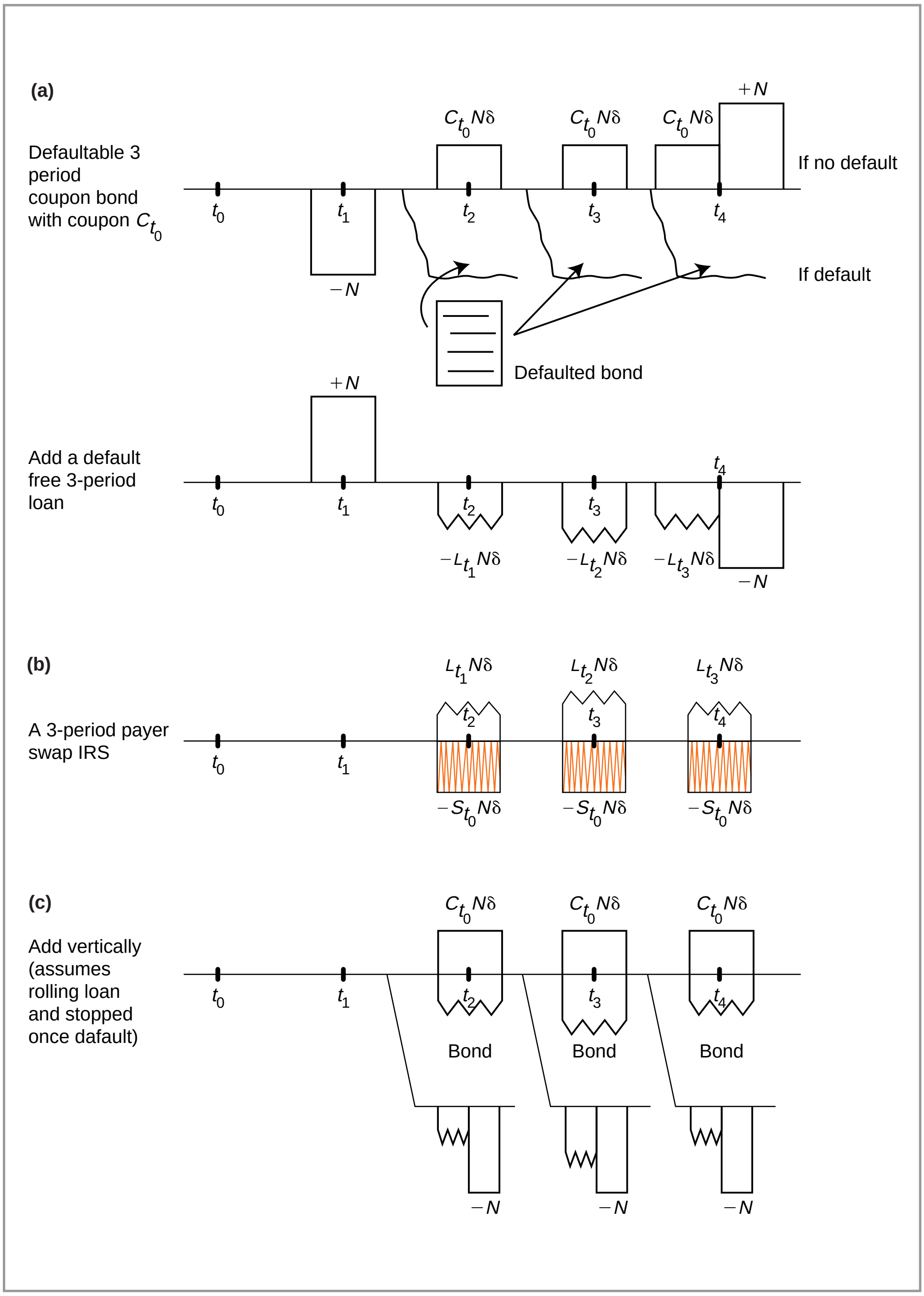


FIGURE 5-5

3. The Instrument: Swaps

Imagine *any* two sequences of cash flows with different characteristics. These cash flows could be generated by any process—a financial instrument, a productive activity, a natural phenomenon. They will also depend on different risk factors. Then one can, in principle, devise a contract

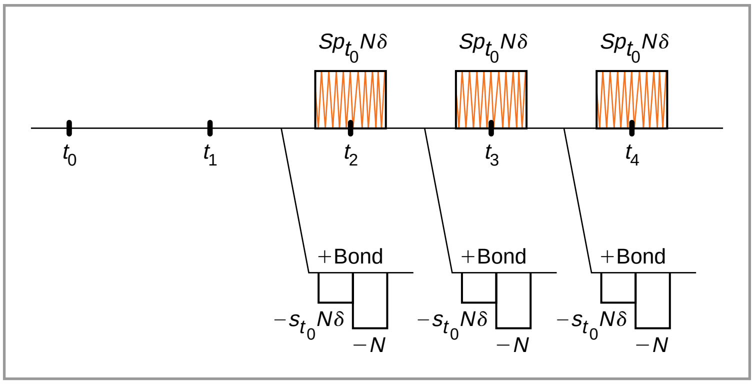


FIGURE 5-6

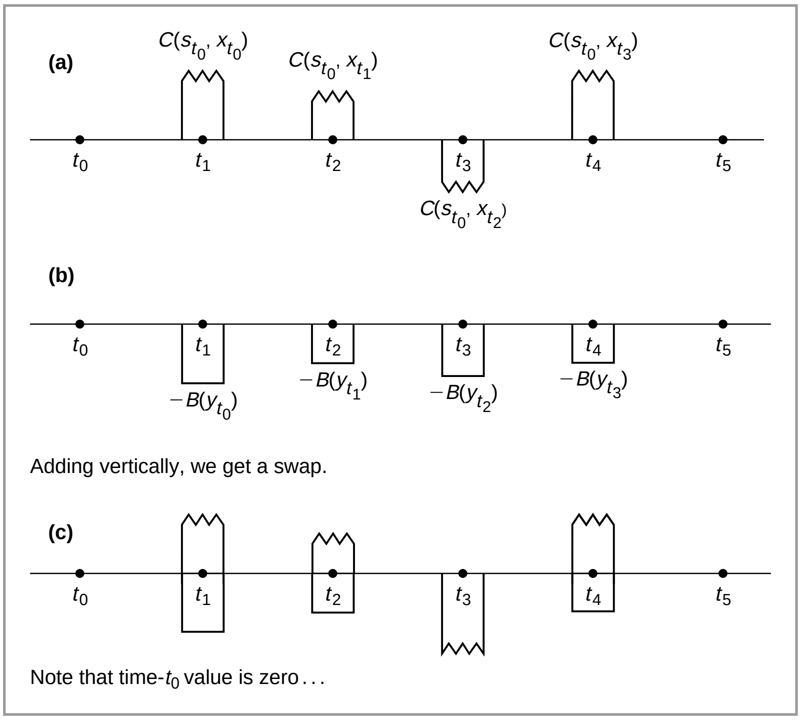


FIGURE 5-7

where these two cash flow sequences are exchanged. This contract will be called a *swap*. To design a swap, we use the following principles:

1. A swap is arranged as a pure exchange of cash flows and hence should not require any additional net cash payments at initiation. In other words, the *initial* value of the swap contract should be *zero*.
2. The contract specifies a *swap spread*. This variable is adjusted to make the two counterparties willing to exchange the cash flows.

A generic exchange is shown in Figure 5-7. In this figure, the first sequence of cash flows starts at time t_1 and continues periodically at $t_2, t_3, \ldots t_k$. There are k *floating* cash flows of differing sizes denoted by

$$\{C(s_{t_0}, x_{t_1}), C(s_{t_0}, x_{t_2}), \ldots, C(s_{t_0}, x_{t_k})\} \tag{19}$$

These cash flows depend on a vector of market or credit risk factors denoted by x_{t_i}. The cash flows depend also on the s_{t_0}, a swap *spread* or an appropriate *swap rate*. By selecting the value of s_{t_0}, the initial value of the swap can be made zero.
Figure 5-6b represents another strip of cash flows:

$$\{B(y_{t_0}), B(y_{t_1}), B(y_{t_2}), \ldots, B(y_{t_k})\} \tag{20}$$

which depend potentially on some other risk factors denoted by y_{t_i}.

The swap consists of exchanging the $\{C(s_{t_0}, x_{t_i})\}$ against$\{B(y_{t_i})\}$ at *settlement dates* $\{t_i\}$. The parameter s_{t_0} is selected at time t_0 so that the two parties are willing to go through with this exchange without any initial cash payment. This is shown in Figure 5-7c. One will *pay* the $C(.)$'s and *receive* the $B(.)$'s. The counterparty will be the "other side" of the deal and will do the reverse.[7] Clearly, if the cash flows are in the *same* currency, there is no need to make two different payments in each period t_i. One party can simply pay the other the *net* amount. Then actual wire transfers will look more like the cash flows in Figure 5-8. Of course, what one party receives is equal to what the counterparty pays.

Now, if two parties who are willing to exchange the two sequences of cash flows without any up-front payment, the market value of these cash flows must be the same no matter how different they are in terms of implicit risks. Otherwise one of the parties will require an up-front net payment. Yet, as time passes, a swap agreement may end up having a positive or negative net value, since the variables x_{t_i} and y_{t_i} will change, and this will make one cash flow more "valuable" than the other.

EXAMPLE:

Suppose you signed a swap contract that entitles you to a 7% return in dollars, in return for a 6% return in Euros. The exchanges will be made every 3 months at a predetermined *exchange rate e_{t_0}. At initiation time t_0, the net value of the commitment should be zero, given the correct* swap spread. *This means that at time t_0 the market value of the receipts and payments are the same. Yet, after the contract is initiated, USD interest rates may fall relative to European rates. This would make the receipt of 7% USD funds relatively more valuable than the payments in Euro.*

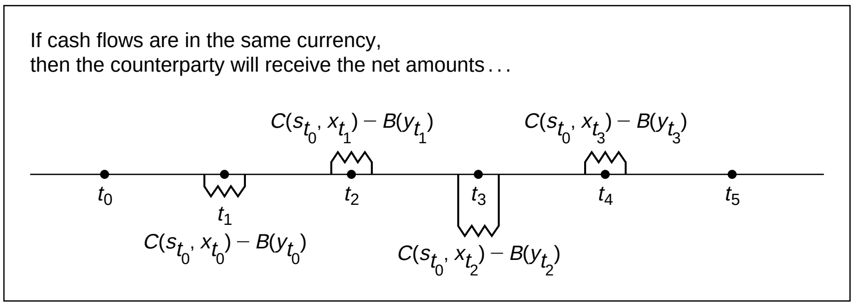


FIGURE 5-8

[7] Here we use the term "cash flows," but it could be that what is exchanged are physical goods.

As a result, from the point of view of the USD-receiving party, the value of the swap will move from zero to positive, while for the counterparty the swap will have a negative value.

Of course, actual exchanges of cash flows at times $t_1, t_2, \ldots, t_n$ may be a more complicated process than the simple transactions shown in Figure 5-8. What exactly is paid or received? Based on which price? Observed when? What are the penalties if deliveries are not made on time? What happens if a t_i falls on a holiday? A typical swap contract needs to clarify many such parameters. These and other issues are specified in the *documentation* set by the International Swaps and Derivatives Association.

4. Types of Swaps

Swaps are a very broad instrument category. Practically, every cash flow sequence can be used to generate a swap. It is impossible to discuss all the relevant material in this book. So, instead of spreading the discussion thinly, we adopt a strategy where a number of critical swap structures are selected and the discussion is centered on these. We hope that the extension of the implied swap engineering to other swap categories will be straightforward.

4.1. *Noninterest Rate Swaps*

Most swaps are interest rate related given the Libor and yield curve exposures on corporate and bank balance sheets. But swaps form a broader category of instruments, and to emphasize this point we start the discussion with noninterest rate swaps. Here the most recent and the most important is the Credit Default Swap. We will examine this *credit* instrument in a separate chapter, and only introduce it briefly here. This chapter will concentrate mainly on two other swap categories: *equity swaps* and *commodity swaps*.

4.1.1. Equity Swaps

Equity swaps exchange equity-based returns against Libor as seen earlier.

In equity swaps, the parties will exchange two sequences of cash flows. One of the cash flow sequences will be generated by dividends and capital gains (losses), while the other will depend on a money market instrument, in general Libor. Once clearly defined, each cash flow can be valued separately. Then, adding or subtracting a *spread* to the corresponding Libor rate would make the two parties willing to exchange these cash flows with no initial payment. The contract that makes this exchange legally binding is called an equity swap.

Thus, a typical equity swap consists of the following. Initiation time will be t_0. An equity index I_{t_i} and a money market rate, say Libor L_{t_i}, are selected. At times $\{t_1, t_2, \ldots, t_n\}$ the parties will exchange cash flows based on the percentage change in I_{t_i}, written as

$$N_{t_{i-1}} \left(\frac{I_{t_i} - I_{t_{i-1}}}{I_{t_{i-1}}} \right) \tag{21}$$

against Libor-based cash flows, $N_{t_{i-1}} L_{t_{i-1}} \delta$ plus or minus a spread. The N_{t_i} is the notional amount, which is not exchanged.

Note that the notional amount is allowed to be reset at every $t_0, t_1, \ldots, t_{n-1}$, allowing the parties to adjust their position in the particular equity index periodically. In equity swaps, this notional principal can also be selected as a constant, N.

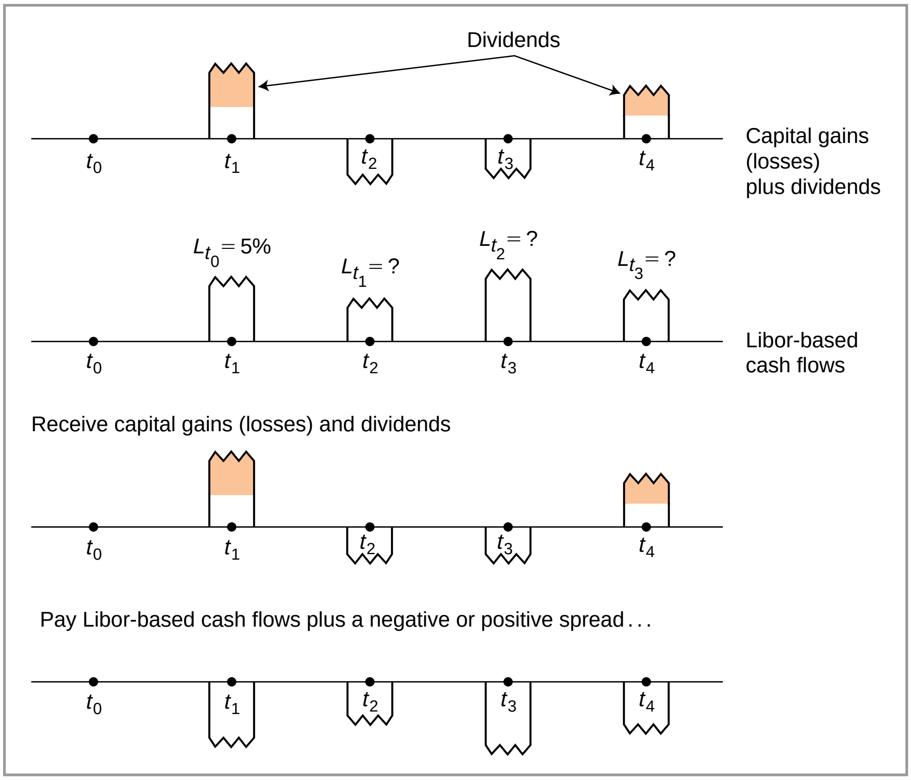


FIGURE 5-9

EXAMPLE:

In Figure 5-9 we have a 4-year sequence of capital gains (losses) plus dividends generated by a certain equity index. They are exchanged every 90 days, against a sequence of cash flows based on 3-month Libor-20 bp.

The notional principal *is USD1 million. At time t_0 the elements of these cash flows will be unknown.*

At time t_1, the respective payments can be calculated once the index performance is observed. Suppose we have the following data:

$$I_{t_0} = 800 \tag{22}$$

$$I_{t_1} = 850 \qquad L_{t_0} = 5\% \qquad \text{spread} = .20 \tag{23}$$

Then the time-t_1 equity-linked cash flow is

$$1m\left(\frac{I_{t_1} - I_{t_0}}{I_{t_0}}\right) = 1{,}000{,}000(0.0625) = 62{,}500 \tag{24}$$

The Libor-linked cash flows will be

$$1m(L_{t_0} - s_{t_0})\frac{90}{360} = 1{,}000{,}000(.05 - .002)\frac{1}{4} = 12{,}000 \tag{25}$$

The remaining unknown cash flows will become known as time passes, dividends are paid, and prices move. The spread is subtracted from the interest rate.

Some equity swaps are between *two* equity indices. The following example illustrates the idea.

EXAMPLE:

In an equity swap, the holder of the instrument pays the total return of the S&P 500 and receives the return on another index, say the Nikkei. Its advantage for the holder lies in the fact that, as a swap, it does not involve paying any up-front premium.

Of course, the same trade could also be created by selling S&P futures and buying futures on another equity index. But the equity swap has the benefit that it simplifies tracking the indices.

Later in this chapter, we will discuss several uses of equity swaps.

4.1.2. Commodity Swaps

The overall structure of commodity swaps is similar to equity swaps. As with equity swaps, there are two major types of commodity swaps. Parties to the swap can, either (1) exchange fixed to floating payments based on a commodity index or, (2) exchange payments when one payment is based on an index and the other on a money market reference rate.

Consider a refinery, for example. Refineries buy crude oil and sell refined products. They may find it useful to lock in a fixed price for crude oil. This way, they can plan future operations better. Hence, using a swap, a refiner may want to receive a floating price of oil and pay a fixed price per barrel.

Such *commodity* or *oil swaps* can be arranged for all sorts of commodities, metals, precious metals, and energy prices.

EXAMPLE:

Japanese oil companies and trading houses are naturally short in crude oil and long in oil products. They use the short-term swap market to cover this exposure and to speculate, through the use of floating/fixed-priced swaps. Due to an over-capacity of heavy oil refineries in the country, the Japanese are long in heavy-oil products and short in light-oil products. This has produced a swap market of Singapore light-oil products against Japanese heavy-oil products.

There is also a "paper balance" market, which is mainly based in Singapore but developing in Tokyo. This is an oil instrument, which is settled in cash rather than through physical delivery of oil. (IFR, Issue 946)

The idea is similar to equity swaps, so we prefer to delay further discussion of commodity swaps until we present the exercises at the end of this chapter.

4.1.3. Credit Swaps

This is an important class of swaps, and it is getting more important by the day. There are many variants of credit swaps, and they will be discussed in more detail in a separate chapter. Here we briefly introduce the main idea, which follows the same principle as other swaps. The *credit default swap* is the main tool for swapping credit. We discuss it briefly in this chapter.

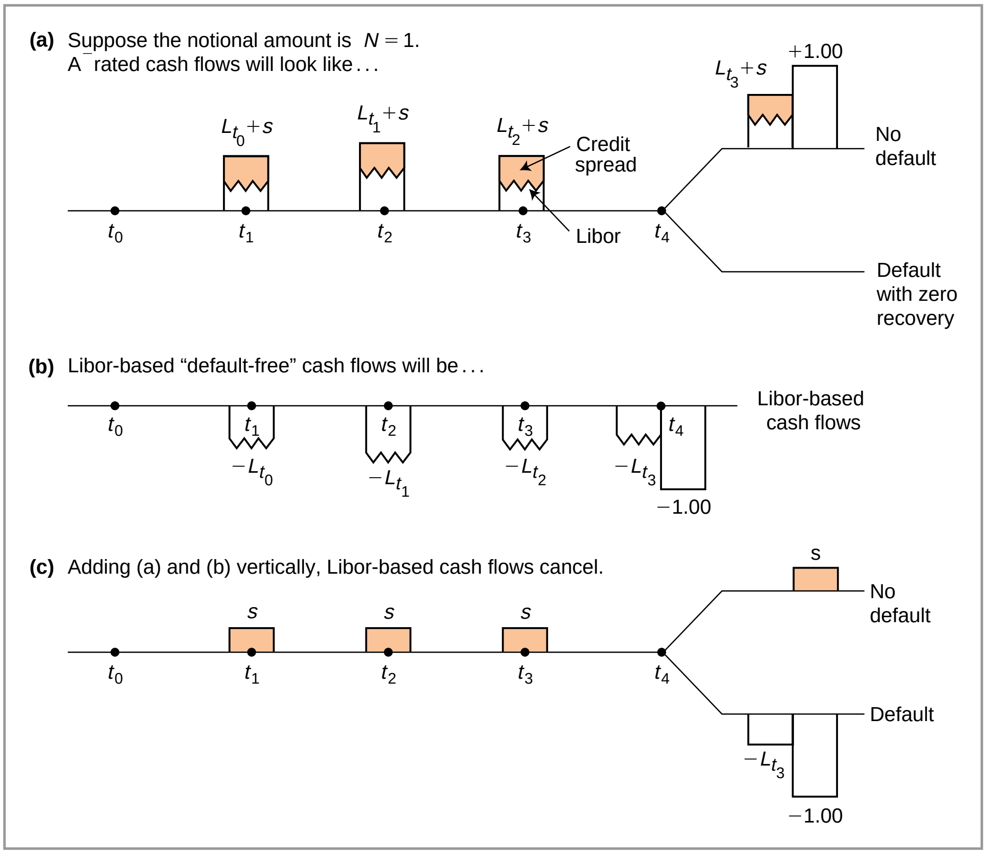


FIGURE 5-10

If swaps are exchanges of cash flows that have different characteristics, then we can consider two sequences of cash flows that are tied to two different *credits*.

A 4-year floating rate cash flow made of Libor *plus* a credit spread is shown in Figure 5-10. The principal is USD1 million and it generates a random cash flow. But there is a critical difference here relative to the previous examples. Since the company may default, there is no guarantee that the interest or the principal will be paid back at future dates. Figure 5-10a simplifies this by assuming that the only possible default on the principal is at time t_4, and that when the default occurs all the principal and interest is lost.[8] Figure 5-10b displays a default-free market cash flow based on 6-month Libor.[9]

By adding the two sequences of cash flows in this example vertically, we get the *credit default swap*, shown in Figure 5-10.

4.2. *Interest Rate Swaps*

This is the largest swap market. It involves exchanging cash flows generated by different interest rates. The most common case is when a *fixed* swap rate is paid (received) against a floating Libor

[8] This means there is no recovery value.

[9] Of course, instead of this, we could use another credit-based cash flow as in Figure 5-10a, but this time with a bond issued by, say, a BB-rated entity.

rate in the *same* currency. Interest rate swaps have become a fundamental instrument in world financial markets. The following reading illustrates this for the case of plain vanilla interest rate swaps.

EXAMPLE:

The swap curve is being widely touted as the best alternative to a dwindling Treasury market for benchmarking U.S. corporate bonds. . . . This has prompted renewed predictions that the swap curve will be adopted as a primary benchmark for corporate bonds and asset-backed securities.

. . . Investors in corporate bonds say there are definite benefits from the increasing attention being paid to swap spreads for valuing bonds. One is that the mortgage-backed securities market has already to a large degree made the shift to use of Libor-based valuation of positions, and that comparability of corporate bonds with mortgage holdings is desirable.

. . . Swap dealers also point out that while the agency debt market is being adroitly positioned by Fannie Mae and Freddie Mac as an alternative to the Treasury market for benchmarking purposes, agency spreads are still effectively bound to move in line with swap spreads.

. . . Bankers and investors agree that hedging of corporate bond positions in the future will effectively mean making the best use of whatever tools are available. So even if swaps and agency bonds have limitations, and credit costs edge up, they will still be increasingly widely used for hedging purposes. (IFR, Issue 6321)

This reading illustrates the crucial position held by the swap market in the world of finance. The "swap curve" obtained from *interest rate swaps* is considered by many as a benchmark for the term structure of interest rates, and this means that most assets could eventually be priced off the interest rate swaps, in one way or another. Also, the reading correctly points out some major sectors in markets. In particular, (1) the mortgage-backed securities (MBS) market, (2) the market for "agencies," which means securities issued by Fannie Mae or Freddie Mac, etc., and (3) the corporate bond market have their own complications, yet, swaps play a major role in all of them. At this point, it is best to define formally the interest rate swap and then look at an example.

A *plain vanilla interest rate swap* (IRS) initiated at time t_0 is a commitment to exchange interest payments associated with a notional amount N, settled at clearly identified settlement dates, $\{t_1, t_2, \ldots, t_n\}$. The *buyer* of the swap will make fixed payments of size $s_{t_0} N\delta$ each and receive floating payments of size $L_{t_i} N\delta$. The Libor rate L_{t_i} is determined at *set dates* $\{t_0, t_1, \ldots, t_{n-1}\}$. The maturity of swap is m years.[10] The s_{t_0} is the swap rate.

EXAMPLE:

An Interest Rate Swap has a notional amount N of USD1 million, a 7% fixed rate for 2 years in semiannual (s.a.) payments against a cash flow generated by 6-month Libor. This is shown in Figure 5-11a. There are two sequences of cash flows. One involves four payments of USD35,000 each. They are known at t_0 *and paid at the* end *of each* 6-month *period.*

The second is shown in Figure 5-11b. These cash flows are determined by the value of 6-month USD Libor to be observed at set dates. Four separate Libor rates will be observed

[10] Here, $m = n\delta$. The δ is the days count parameter.

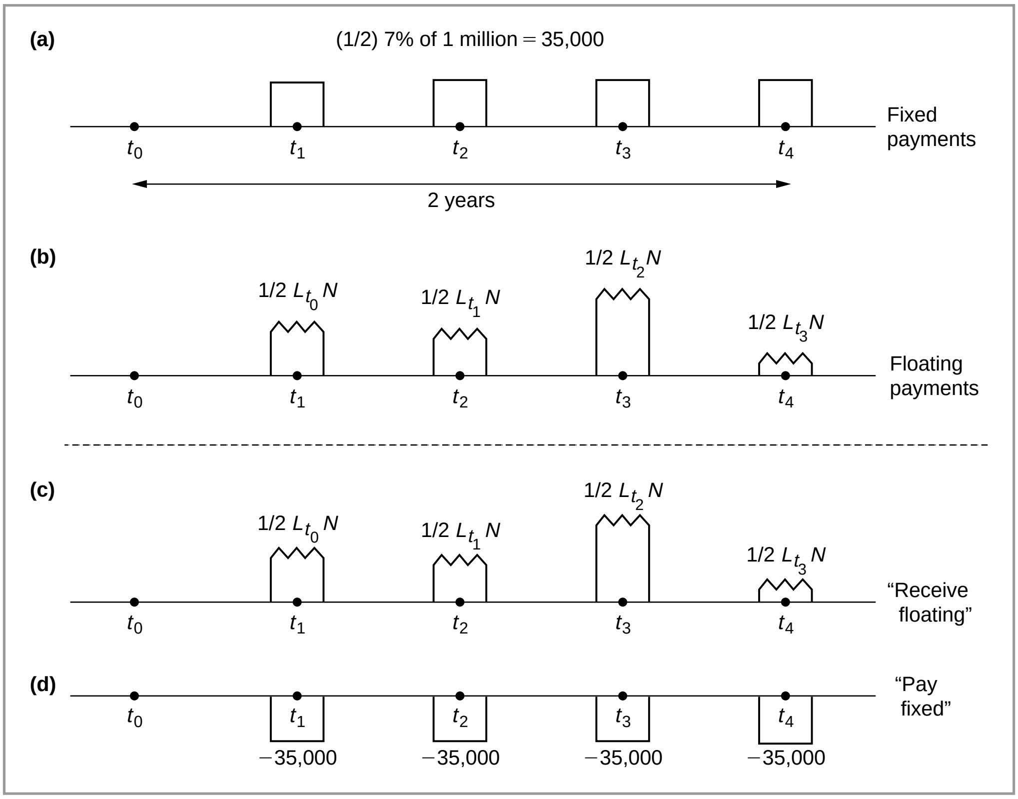


FIGURE 5-11

during this period. The L_{t_0} is known at the initial point t_0. The remaining Libor rates, L_{t_1}, L_{t_2}, and L_{t_3}, will be observed gradually as time passes but are unknown initially.

In Figure 5-11, the floating cash flows depending on L_{t_i} are observed at time t_i, but are *paid-in-arrears* at times t_{i+1}. Swaps that have this characteristic are known as paid-in-arrears swaps.

Clearly, we have two sets of cash flows with different market risk characteristics. The market will price them separately. Once this is done, market participants can trade them. A *fixed payer* will pay the cash flows in Figure 5-11a and receive the one in Figure 5-11b. This institution is the *buyer* of the interest rate swap.

The market participant on the other side of the deal will be doing the reverse—receiving cash flows based on a fixed interest rate at time t_0, while paying cash flows that become gradually known as time passes and the Libor rates L_{t_i} are revealed. This party is the *fixed receiver*, whom the market also calls the *seller* of the swap.[11] We can always make the exchange of the two cash flows acceptable to both parties by adding a proper spread to one of the cash flows.[12] This role is played by the swap spread. The market includes the spread in the fixed rate. By adjusting this spread accordingly, the two parties may be brought together and accept the exchange of one

[11] Similarly to the FRA terminology, those who pay a fixed rate are in general players who are looking to lock in a certain interest rate and reduce risks associated with floating rates. These are clients who need "protection." Hence, it is said that they are *buying* the swap.

[12] After all, apples and oranges are rarely traded one to one.

cash flow against another. The agreed fixed rate is the *swap rate*. We have:

$$\textit{Swap rate} = \textit{Benchmark rate} + \textit{Swap spread} \tag{26}$$

The *benchmark rate* is often selected as the same maturity sovereign bond in that currency.

The final cash flows of an interest rate swap from the fixed payer's point of view will be as shown in Figure 5-8. Only the net amount will change hands.

A real-life example might be helpful. In the following, we consider a private company that is contemplating an increase in the proportion of its floating rate debt. The company can do this by issuing short-term paper, called commercial paper (CP), and continuing to roll over the debt when these obligations mature. But a second way of doing it is by first issuing a 5-year fixed-rate bond, and then swapping the interest paid into floating interest rates.

EXAMPLE:

A corporation considers issuing commercial paper or a medium-term fixed-rate bond (MTN) that it can convert to a floating-rate liability via a swap. The company is looking to increase the share of floating-rate liabilities to 50%–55% from 30%.

The alternative to tapping the MTN market is drawing on its $700 million commercial paper facility.

This reading shows one role played by swaps in daily decisions faced by corporate treasuries. The existence of swaps makes the rates observed in the important CP-sector more closely related to the interest rates in the MTN-sector.

4.2.1. Currency Swaps

Currency swaps are similar to interest rate swaps, but there are some differences. First, the exchanged cash flows are in *different currencies*. This means that two different yield curves are involved in swap pricing instead of just one. Second, in the large majority of cases a floating rate is exchanged against another floating rate. A third difference lies in the exchange of principals at initiation and a re-exchange at maturity. In the case of interest rate swaps this question does not arise since the notional amounts are in the same currency. Currency swaps can be engineered almost the same way as interest rate swaps.

Formally, a currency swap will have the following components. There will be two currencies, say USD($) and Euro(€). The swap is initiated at time t_0 and involves (1) an exchange of a principal amount $N^\$$ against the principal $M^€$ and (2) a series of floating interest payments associated with the principals $N^\$$ and $M^€$, respectively. They are settled at settlement dates, $\{t_1, t_2, \ldots, t_n\}$. One party will pay the floating payments $L^\$_{t_i} N^\$ \delta$ and receive floating payments of size $L^€_{t_i} M^€ \delta$. The *two* Libor rates $L^\$_{t_i}$ and $L^€_{t_i}$ will be determined at *set dates* $\{t_0, t_1, \ldots, t_{n-1}\}$. The maturity of swap will be m years.

A small *spread* s_{t_0} can be added to one of the interest rates to make both parties willing to exchange the cash flows. The market maker will quote bid/ask rates for this spread.

EXAMPLE:

Figure 5-12 shows a currency swap. The USD notional amount is 1 million. The current USD/EUR exchange rate is at .95. The agreed spread is 6 bp. The initial 3-month Libor rates are

$$L^\$_{t_i} = 3\% \tag{27}$$

$$L^€_{t_i} = 3.5\% \tag{28}$$

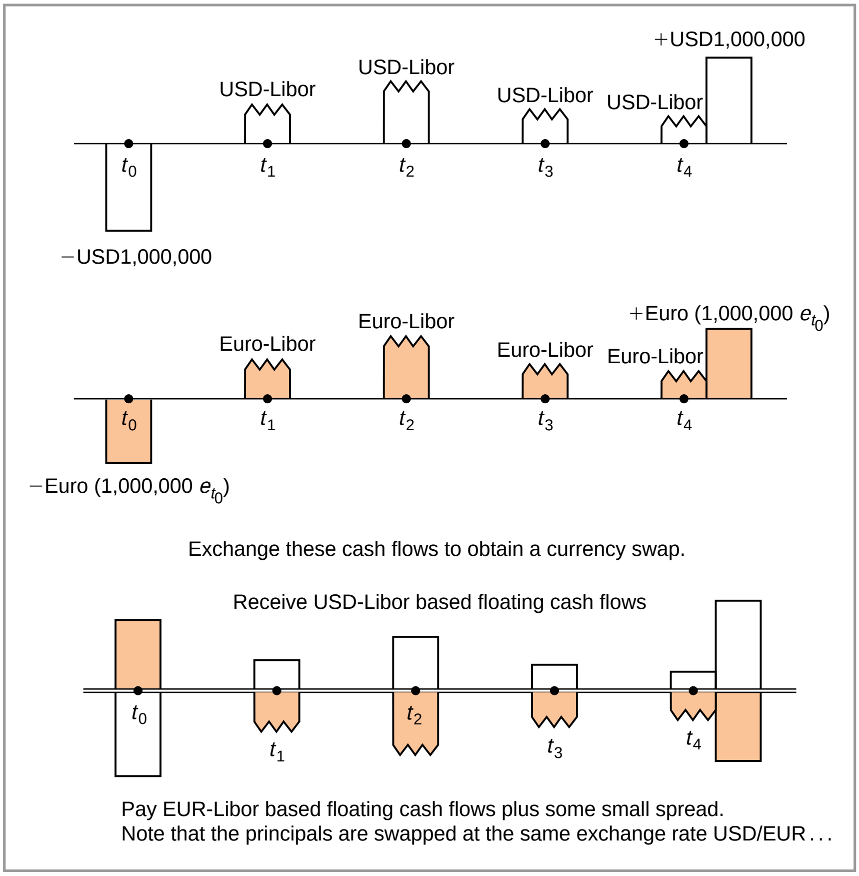


FIGURE 5-12

This means that at the first settlement date

$$(1{,}000{,}000)(.03 + .0006)\frac{1}{4} = \$7{,}650 \tag{29}$$

will be exchanged against

$$0.95(1{,}000{,}000)(.035)\frac{1}{4} = \text{€}\, 8312.5 \tag{30}$$

All other interest payments would be unknown. Note that the Euro principal amount is related to the USD principal amount according to

$$e_{t_0} N^{\$} = M^{\text{€}} \tag{31}$$

where e_{t_0} *is the spot exchange rate at* t_0.

Also, note that we created the swap spread to the USD Libor.

Pricing currency swaps will follow the same principles as interest rate swaps. A currency swap involves well-defined cash flows and consequently we can calculate an arbitrage-free value for each sequence of cash flows. Then these cash flows are traded. An appropriate *spread* is added to either floating rate.

By adjusting this spread, a swap dealer can again make the two parties willing to exchange the two cash flows.

4.2.2. Basis Swaps

Basis swaps are similar to currency swaps except that often there is only one currency involved. A basis swap involves exchanging cash flows in one *floating* rate, against cash flows in another *floating* rate, in the same currency. One of the involved interest rates is often a non-Libor-based rate, and the other is Libor.

The following reading gives an idea about the basis swap. Fannie Mae, a U.S. government *agency*, borrows from international money markets in USD Libor and then lends these funds to mortgage banks. Fannie Mae faces a *basis risk* while doing this. There is a small difference between the interest rate that it eventually pays, which is USD Libor, and the interest rate it eventually receives, the USD discount rate. To hedge its position, Fannie Mae needs to convert one floating rate to the other. This is the topic of the reading that follows:

EXAMPLE:

Merrill Lynch has been using Fannie Mae benchmark bonds to price and hedge its billion dollar discount/basis swap business. "We have used the benchmark bonds as a pricing tool for our discount/Libor basis swaps since the day they were issued. We continue to use them to price the swaps and hedge our exposure," said the head of interest rate derivatives trading.

He added that hedging activity was centered on the five and 10-year bonds—the typical discount/Libor basis swap tenors. Discount/Libor swaps and notes are employed extensively by U.S. agencies, such as Fannie Mae, to hedge their basis risk. They lend at the U.S. discount rate but fund themselves at the Libor rate and as a result are exposed to the Libor/discount rate spread. Under the basis swap, the agency/municipality receives Libor and in return pays the discount rate.

Major U.S. derivative providers began offering discount/Libor basis swaps several years ago and now run billion dollar books. (IFR, Issue 1229)

This reading illustrates two things. Fannie Mae needs to swap one floating rate to another in order to allow the receipts and payments to be based on the same risk. But at the same time, *because* Fannie Mae is hedging using basis swaps and because there is a large amount of such Fannie Mae bonds, some market practitioners may think that these *agency bonds* make good pricing tools for basis swaps themselves.

4.2.3. What Is an Asset Swap?

The term *asset swap* can, in principle, be used for any type of swap. After all, sequences of cash flows considered thus far are generated by some assets, indices, or reference rates. Also, swaps linked to equity indices or reference rates such as Libor can easily be visualized as Floating Rate Notes (FRN), corporate bond portfolios, or portfolios of stocks. Exchanging these cash flows is equivalent to exchanging the underlying asset.

Yet, the term asset swap is often used with a more precise meaning. Consider a defaultable *par* bond that pays annual coupon c_{t_0}. Suppose the payments are semi-annual. Then we can imagine a swap where coupon payments are exchanged against 6-month Libor L_{t_i} plus a spread s_{t_0}, every 6 months. The coupon payments are fixed and known at t_0. The floating payments will be random, although the spread component, s_{t_0}, is known at time t_0 as well. This structure is often labelled an asset swap.[13] The reader can easily put together the cash flows implied by

[13] In an asset swap credit risk remains with the bond holder.

this instrument, if the issue of default is ignored. Such a cash flow diagram would follow the exchanges shown in Figure 5-7. One sequence of cash flows would represent coupons, the other Libor plus a spread.

Asset swaps interpreted this way offer a useful alternative to investors. An investor can always buy a bond and receive the coupon c_{t_0}. But by using an asset swap, the investor can also swap out of the coupon payments and receive only floating Libor plus the spread s_{t_0}. This way the exposure to the *issuer* is kept and the exposure to *fixed interest rates* is eliminated. In fact, treasury bonds or fixed receiver interest swaps may be better choices if one desires exposure to fixed interest rates. Given the use of Libor in this structure, the s_{t_0} is calculated as the spread to the corresponding fixed swap rates.

4.2.4. More Complex Swaps

The swaps discussed thus far are liquid and are traded actively. One can imagine many other swaps. Some of these are also liquid, others are not. Amortizing swaps, bullion swaps, MBS swaps, quanto (differential) swaps are some that come to mind. We will not elaborate on them at this point; some of these swaps will be introduced as examples or exercises in later chapters.

An interesting special case is constant maturity swaps (CMS), which will be discussed in detail in Chapter 15. The CMS swaps have an interesting convexity dimension that requires taking into account volatilities and correlations across various forward rates along a yield curve. A related swap category is constant maturity treasury (CMT) swaps.

4.3. *Swap Conventions*

Interest rate swap markets have their own conventions. In some economies, the market quotes the swap *spread*. This is the case for USD interest rate swaps. USD interest rates swaps are quoted as a spread to Treasuries. In Australia, the market also quotes swap spreads. But the spreads are to bond futures.

In other economies, the market quotes the swap *rate*. This is the case for Euro interest rate swaps.

Next, there is the issue of how to quote swaps. This is done in terms of two-way interest rate quotes. But sometimes the quoted swap rate is on an annual basis, and sometimes it is on a semiannual basis. Also, the day-count conventions change from one market to another. In USD swaps, the day-count is in general ACT/360. In EUR swaps day-count is 30/360.

According to market conventions, a fixed payer called, *the payer*, is *long* the swap, and has *bought* a swap. On the other hand, a fixed receiver called, *the receiver*, is *short* the swap, and has *sold* a swap.

5. Engineering Interest Rate Swaps

We now study the financial engineering of swaps. We focus on plain vanilla *interest rate* swaps. Engineering of other swaps is similar in many ways, and is left to the reader. For simplicity, we deal with a case of only three settlement dates. Figure 5-13 shows a *fixed-payer*, three-period interest rate swap, with start date t_0. The swap is initiated at date t_0. The party will make three fixed payments and receive three floating payments at dates t_1, t_2, and t_3 in the same currency. The dates t_1, t_2, and t_3 are the settlement dates. The t_0, t_1, and t_2 are also the *reset* dates, dates on which the relevant Libor rate is determined.

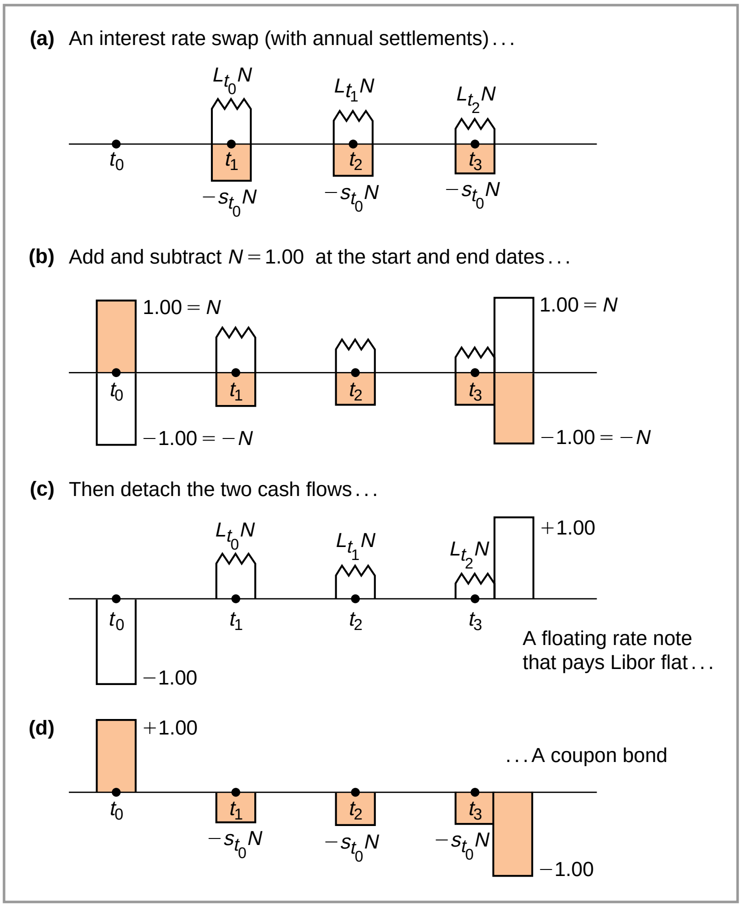


FIGURE 5-13

We select the notional amount N as unity and let $\delta = 1$, assuming that the floating rate is 12-month Libor:[14]

$$N = \$1 \tag{32}$$

Under these simplified conditions the fixed payments equal s_{t_0}, and the Libor-linked payments equal L_{t_0}, L_{t_1}, and L_{t_2}, respectively. The *swap spread* will be the difference between s_{t_0} and the treasury rate on the bond with the same maturity, denoted by y_{t_0}.[15] Thus, we have

$$\text{Swap spread} = s_{t_0} - y_{t_0} \tag{33}$$

We will study the engineering of this interest rate swap. More precisely, we will discuss the way we can replicate this swap. More than the exact synthetic, what is of interest is the way(s) one can approach this problem.

A swap can be reverse-engineered in at least two ways:

1. We can first decompose the swap *horizontally*, into two streams of cash flows, one representing a floating stream of payments (receipts), the other a fixed stream. If this is done, then each stream can be interpreted as being linked to a certain type of bond.

[14] This is a simplification. In reality, the floating rate is either 3-month or 6-month Libor.

[15] This could be any interest rate accepted as a benchmark by the market.

2. Second, we can decompose the swap *vertically*, slicing it into n cash exchanges during n time periods. If this is done, then each cash exchange can be interpreted similarly (but not identically) to a FRA paid-in-arrears, with the property that the fixed rate is constant across various settlement dates.

We now study each method in detail.

5.1. A Horizontal Decomposition

First we simplify the notation and the parameters used in this section. To concentrate on the engineering aspects only, we prefer to eliminate some variables from the discussion. For example, we assume that the swap will make payments every year so that the day-count parameter is $\delta = 1$, unless assumed otherwise. Next, we discuss a *forward* swap that is signed at time t_0, but starts at time t_1, with $t_0 < t_1$. During this discussion, we may occasionally omit the use of the term "forward" and refer to the forward swap simply as a swap.[16]

The traditional way to decompose an interest rate swap is to do this horizontally. The original swap cash flows are shown in Figure 5-13a. Before we start, we need to use a trick. We add and subtract the same notional amount N at the start, and end dates, for both sequences of cash flows. Since these involve identical currencies and identical amounts, they cancel out and we recover the standard exchanges of floating versus fixed-rate payments. With the addition and subtraction of the initial principals, the swap will look as in Figure 5-13b.

Next, "detach" the cash flows in Figure 5-13b horizontally, so as to obtain two separate cash flows as shown in Figures 5-13c and 5-13d. Note that each sequence of cash flows is already in the form of a meaningful financial contract.

In fact, Figure 5-13c can immediately be recognized as representing a long forward position in a floating rate note that pays Libor flat. At time t_1, the initial amount N is paid and L_{t_1} is set. At t_2, the first interest payment is received, and this will continue until time t_4 where the last interest is received along with the principal.

Figure 5-13d can be recognized as a short forward position on a par coupon bond that pays a coupon equal to s_{t_0}. We (short) sell the bond to receive N. At every payment date the fixed coupon is paid and then, at t_4, we pay the last coupon and the principal N.

Thus, the immediate decomposition suggests the following synthetic:

$$\text{Interest rate swap} = \{\text{Long FRN with Libor coupon, short par coupon bond}\} \quad (34)$$

Here the bond in question needs to have the same credit risk as in a flat Libor-based loan. Using this representation, it is straightforward to write the contractual equation:

Long swap	$=$	Short a par bond with coupon s_{t_0}	$+$	Long FRN paying Libor flat	(35)

Using this relationship, one can follow the methodology introduced earlier and immediately generate some interesting synthetics.

[16] Remember also that there is no credit risk, and that time is discrete. Finally, there are no bid-ask spreads.

5.1.1. A Synthetic Coupon Bond

Suppose an AAA-rated entity with negligible default risk issues only 3-year FRNs that pay Libor – 10 bp every 12 months.[17] A client would like to buy a coupon bond from this entity, but it turns out that no such bonds are issued. We can help our client by synthetically creating the bond. To do this, we manipulate the contractual equation so that we have a long coupon bond on the right-hand side:

Long par bond with coupon $s_{t_0} - 10$ bp	=	Sell a swap with rate s_{t_0}	+	Long FRN paying Libor–10 bp	(36)

The geometry of this engineering is shown in Figure 5-14. The synthetic results in a coupon bond issued by the same entity and paying a coupon of s_{t_0} – 10 bp. The 10 bp included in the coupon account for the fact that the security is issued by an AAA-rated entity.

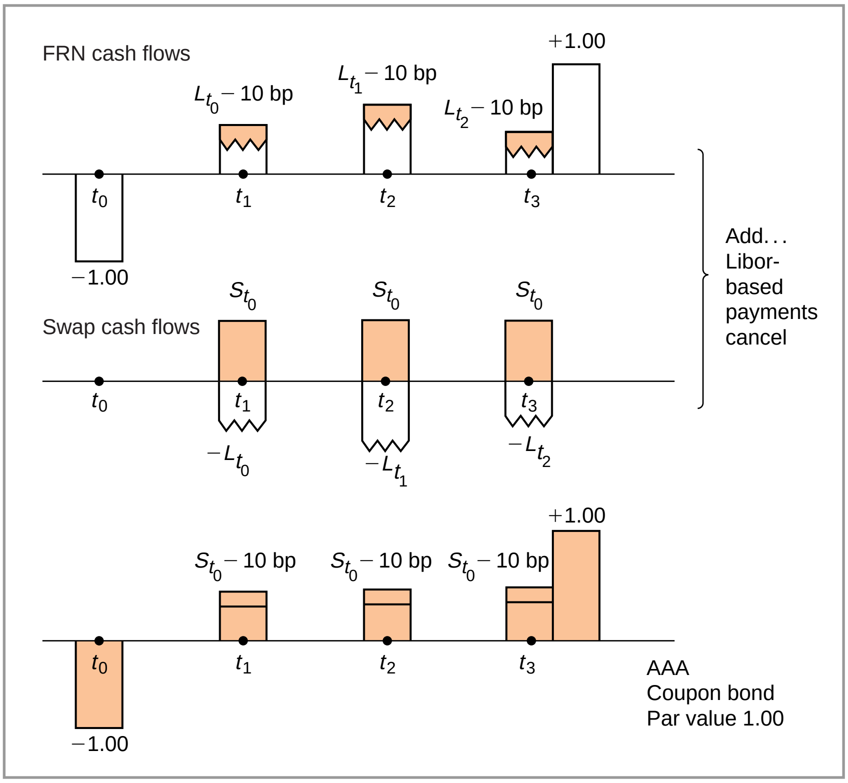


FIGURE 5-14

[17] Before the credit crisis of 2007–2008, AAA entities did have “negligible” credit risk, and spread to Libor was negative.

5.1.2. Pricing

The contractual equation obtained in (35) permits pricing the swap off the *debt markets*, using observed prices of fixed and floating coupon bonds. To see this we write the present value of the cash streams generated by the fixed and floating rate bonds using appropriate discount factors. Throughout this section, we will work with a special case of a 3-year spot swap that makes fixed payments against 12-month Libor. This simplifies the discussion. It is also straightforward to generalize the ensuing formulas to an n-year swap.

Suppose the swap makes three annual coupon payments, each equaling $s_{t_0}N$. We also have three floating rate payments each with the value $L_{t_{i-1}}N$, where the relevant Libor $L_{t_{i-1}}$ is set at t_{i-1}, but is paid at t_i.

5.1.3. Valuing Fixed Cash Flows

To obtain the present value of the fixed cash flows, we discount them by the relevant floating rates. Note that, *if* we knew the floating rates $\{L_{t_0}, L_{t_1}, L_{t_2}\}$, we could write

$$\text{PV-fixed} = \frac{s_{t_0}N}{(1+L_{t_0})} + \frac{s_{t_0}N}{(1+L_{t_0})(1+L_{t_1})} + \frac{s_{t_0}N+N}{(1+L_{t_0})(1+L_{t_1})(1+L_{t_2})} \tag{37}$$

where we added N to date t_3 cash flows. But at $t=0$, the Libor rates $L_{t_i}, i=1,2$ are unknown. Yet, we know that against each $L_{t_i}, i=1,2$, the market is willing to pay the known forward, (FRA) rate, $F(t_0,t_i)$. Thus, using the FRA rates *as if* they are the time-t_0 market values of the unknown Libor rates, we get

$$\text{PV-fixed as of } t_0 = \frac{s_{t_0}N}{(1+F(t_0,t_0))} + \frac{s_{t_0}N}{(1+F(t_0,t_0))(1+F(t_0,t_1))} \tag{38}$$

$$+ \frac{s_{t_0}N+N}{(1+F(t_0,t_0))(1+F(t_0,t_1))(1+F(t_0,t_2))} \tag{39}$$

All the right-hand side quantities are known, and the present value can be calculated exactly, given the s_{t_0}.

5.1.4. Valuing Floating Cash Flows

For the floating rate cash flows we have[18]

$$\text{PV-floating as of } t_0 = \frac{L_{t_0}N}{(1+L_{t_0})} + \frac{L_{t_1}N}{(1+L_{t_0})(1+L_{t_1})} + \frac{L_{t_2}N+N}{(1+L_{t_0})(1+L_{t_1})(1+L_{t_2})} \tag{40}$$

Here, to get a numerical answer, we don't even need to use the forward rates. This present value can be written in a much simpler fashion, once we realize the following transformation:

$$\frac{L_{t_2}N+N}{(1+L_{t_0})(1+L_{t_1})(1+L_{t_2})} = \frac{(1+L_{t_2})N}{(1+L_{t_0})(1+L_{t_1})(1+L_{t_2})} \tag{41}$$

$$= \frac{N}{(1+L_{t_0})(1+L_{t_1})} \tag{42}$$

[18] We remind the reader that the day's adjustment factor was selected as $\delta = 1$ to simplify the exposition. Otherwise, all Libor rates and forward rates in the formulas will have to be multiplied by the δ.

Then, add this to the second term on the right-hand side of the present value in (40) and use the same simplification,

$$\frac{L_{t_1}N}{(1+L_{t_0})(1+L_{t_1})} + \frac{N}{(1+L_{t_0})(1+L_{t_1})} = \frac{N}{(1+L_{t_0})} \tag{43}$$

Finally, apply the same trick to the first term on the right-hand side of (40) and obtain, somewhat surprisingly, the expression

$$\text{PV of floating payments as of } t_0 = N \tag{44}$$

According to this, the present value of a FRN equals the par value N at *every settlement date*. Such recursive simplifications can be applied to present values of floating rate payments at reset dates.[19] We can now combine these by letting

$$\text{PV of fixed payments as of } t_0 = \text{ PV of floating payments as of } t_0 \tag{45}$$

This gives an equation where s_{t_0} can be considered as an unknown:

$$\frac{s_{t_0}N}{(1+F(t_0,t_0))} + \frac{s_{t_0}N}{(1+F(t_0,t_0))(1+F(t_0,t_1))} + \frac{s_{t_0}N+N}{(1+F(t_0,t_0))(1+F(t_0,t_1))(1+F(t_0,t_2))} = N \tag{46}$$

Cancelling the N and rearranging, we can obtain the numerical value of s_{t_0} given $F(t_0,t_0)$, $F(t_0,t_1)$, and $F(t_0,t_2)$. This would value the swap off the FRA markets.

5.1.5. An Important Remark

Note a very convenient, but *very delicate* operation that was used in the preceding derivation. Using the liquid FRA markets, we "replaced" the unknown L_{t_i} by the known $F(t_0,t_i)$ in the appropriate formulas. Yet, these formulas were nonlinear in L_{t_i} and even if the forward rate is an unbiased forecast of the appropriate Libor,

$$F(t_0,t_i) = E^{P^*}_{t_0}\left[L_{t_i}\right] \tag{47}$$

under some appropriate probability P^*, it is not clear whether the substitution is justifiable. For example, it is known that the conditional expectation operator at time t_0, represented by $E^{P^*}_{t_0}[.]$, cannot be moved *inside* a nonlinear formula due to Jensen's inequality:

$$E^{P^*}_{t_0}\left[\frac{1}{1+L_{t_i}\delta}\right] > \frac{1}{1+E^{P^*}_{t_0}[L_{t_i}]\delta}. \tag{48}$$

So, it is not clear how L_{t_i} can be replaced by the corresponding $F(t_0,t_i)$, even when the relation in (47) is true. These questions will have to be discussed after the introduction of risk-neutral and forward measures in Chapters 11 and 12. Such "substitutions" are delicate and depend on many conditions. In our case we are allowed to make the substitution, because the forward rate is what market "pays" for the corresponding Libor rates at time t_0.

[19] Of course, this result does not hold *between* reset dates since the numerator and the denominator terms will, in general, be different. The payments will be made according to $(1+L_{t_i})$, but the present values will use the observed Libor *since the last reset date*:

$$(1+L_{t_0+u}\delta)(1+L_{t_1})$$

where u is the time elapsed since the last reset date. The L_{t_0+u} is the rate observed at time t_0+u. The value of the cash flow will be a bit greater or smaller, depending on the value of L_{t_0+u}.

5.2. A Vertical Decomposition

We now study the second way of decomposing the swap. We already know what FRA contracts are. Consider an annual FRA where the $\delta = 1$. Also, let the FRA be paid-in-arrear. Then, at some time $t_i + \delta$, the FRA parties will exchange the cash flow:

$$(L_{t_i} - F(t_0, t_i))\delta N, \tag{49}$$

where N is the notional amount, $\delta = 1$, and the $F(t_0, t_i)$ is the FRA rate determined at time t_0. We also know that the FRA amounts to exchanging the fixed payment $F(t_0, t_i)\delta N$ against the floating payment $L_{t_i}\delta N$.

Is it possible to decompose a swap into n FRAs, each with a FRA rate $F(t_0, t_i), i = 1, . . . , n$? The situation is shown in Figure 5-15 for the case $n = 3$. The swap cash flows are split by slicing the swap vertically at each payment date. Figures 5-15b, 5-15c, and 5-15d

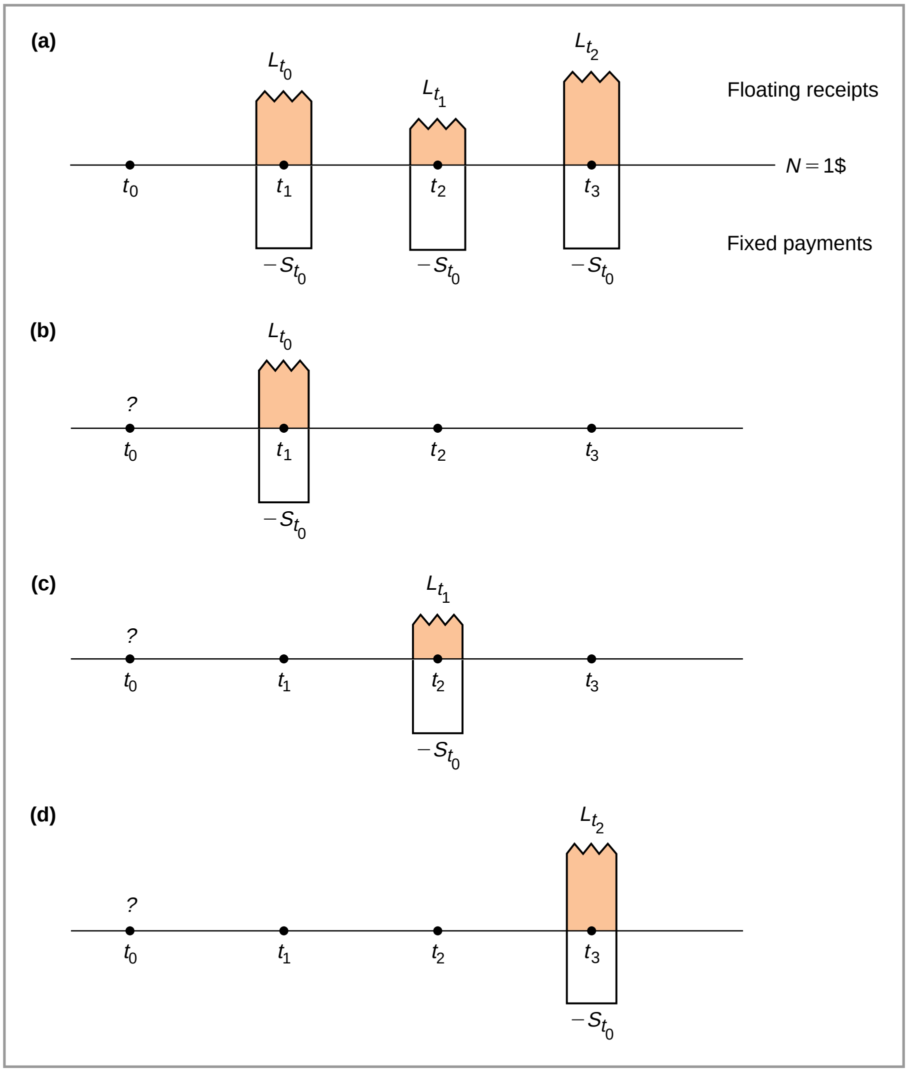


FIGURE 5-15

represent each swap cash flow separately. A *fixed* payment is made against an unknown floating Libor rate, in each case.

Are the cash exchanges shown in Figures 5-15b, 5-15c, and 5-15d tradeable contracts? In particular, are they valid FRA contracts, so that the swap becomes a portfolio of three FRAs? At first glance, the cash flows indeed look like FRAs. But when we analyze this claim more closely, we see that these cash flows are not valid contracts individually.

To understand this, consider the time-t_2 settlement in Figure 5-15b together with the FRA cash flows for the same settlement date, as shown in Figure 5-16. This figure displays an important phenomenon concerning cash flow analysis. Consider the FRA cash flows initiated at time t_0 and settled in arrears at time t_2 and compare these with the corresponding swap settlement. The two cash flows look similar. A fixed rate is exchanged the same against Libor rate L_{t_1} observed at time t_1. But there is still an important difference.

First of all, note that the FRA rate $F(t_0, t_i)$ is determined by supply and demand or by pricing through money markets. Thus, in general

$$F(t_0, t_i) \neq s_{t_0} \tag{50}$$

This means that if we buy the cash flow in Figure 5-16a, and sell the cash flow in Figure 5-16b, Libor-based cash flows will cancel out, but the fixed payments won't. As a result, the portfolio will have a *known* negative or positive net cash flow at time t_2, as shown in Figure 5-16c. Since this cash flow is known exactly, the present value of this portfolio *cannot* be zero. But the present value of the FRA cash flow *is* zero, since (newly initiated) FRA contracts do not have any initial cash payments. All these imply that the time-t_2 cash flow shown in Figure 5-16c will have a known present value,

$$B(t_0, t_2)[F(t_0, t_1) - s_{t_0}]\delta N, \tag{51}$$

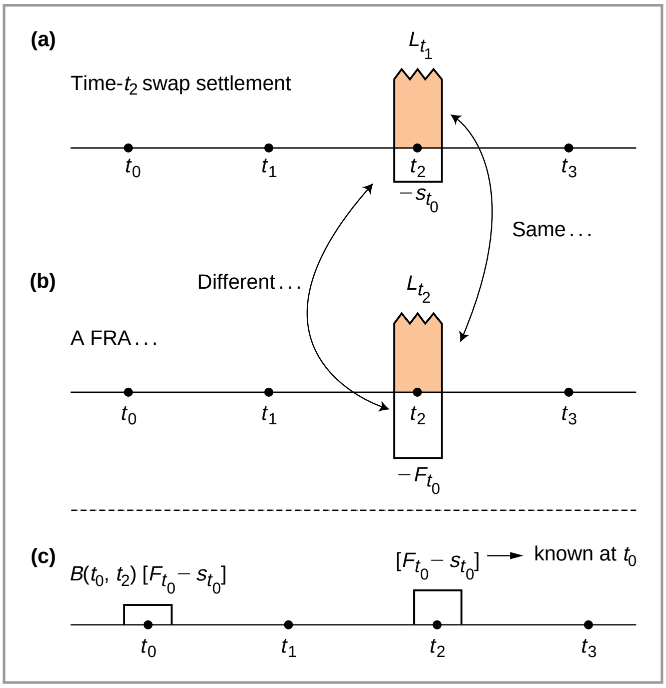


FIGURE 5-16

where the $B(t_0, t_2)$ is the time-t_0 value of the default-free zero coupon bond that matures at time t_2, with par value USD1. This present value will be positive or negative depending on whether $F(t_0, t_1) < s_{t_0}$ or not.

Hence, slicing the swap contract vertically into separate FRA-like cash exchanges does *not* result in tradeable financial contracts. In fact, the time-t_2 exchange shown in Figure 5-16c has a missing time t_0 cash flow of size $B(t_0, t_2)[F(t_0, t_1) - s_{t_0}]\delta N$. Only by adding this, does the exchange become a tradeable contract. The s_{t_0} is a fair exchange for the risks associated with L_{t_0}, L_{t_1}, and L_{t_2} *on the average*. As a result, the time-t_2 cash exchange that is part of the swap contract ceases to have a zero present value when considered individually.

5.2.1. Pricing

We have seen that it is not possible to interpret the individual swap settlements as FRA contracts directly. The two exchanges have a nonzero present value, while the (newly initiated) FRA contracts have a price of zero. But the previous analysis is still useful for pricing the swap since it gives us an important relationship.

In fact, we just showed that the time-t_2 element of the swap has the present value $B(t_0, t_2)$ $[F(t_0, t_1) - s_{t_0}]\delta N$, which is not, normally, zero. This must be true for all swap settlements individually. Yet, the swap cash flows *altogether* do have zero present value. This leads to the following important pricing relation:

$$B(t_0,t_1)[F(t_0,t_0)-s_{t_0}]\delta N+B(t_0,t_2)[F(t_0,t_1)-s_{t_0}]\delta N+B(t_0,t_3)[F(t_0,t_2)-s_{t_0}]\delta N=0 \tag{52}$$

Rearranging provides a formula that ties the swap rate to FRA rates:

$$s_{t_0} = \frac{B(t_0,t_1)F(t_0,t_0) + B(t_0,t_2)F(t_0,t_1) + B(t_0,t_3)F(t_0,t_2)}{B(t_0,t_1) + B(t_0,t_2) + B(t_0,t_3)} \tag{53}$$

This means that we can price swaps off the FRA market as well. The general formula, where n is the *maturity* of the swap,

$$s_{t_0} = \frac{\sum_{i=1}^{n} B(t_0,t_i)F(t_0,t_{i-1})}{\sum_{i=1}^{n} B(t_0,t_i)} \tag{54}$$

will be used routinely in this book. It is an important arbitrage relationship between swap rates and FRA rates.

6. Uses of Swaps

The general idea involving the use of swaps in financial engineering is easy to summarize. A swap involves exchanges of cash flows. But cash flows are generated by assets, liabilities, and other commitments. This means that swaps are simply a standardized, liquid, and cost-effective alternative to trading cash assets. Instead of trading the cash-asset or liability, we are simply trading the cash flows generated by it. Because swaps, in general, have zero value at the time of initiation, and are very liquid, this will indeed be a cost-effective alternative—hence their use in position taking, hedging, and risk management. What are these uses of swaps? We begin the discussion by looking at equity swaps. We will see that these swaps have convenient balance sheet implications, as seen for the FX-swap in Chapter 3. Regulatory and tax treatment of equity swaps are also relevant.

6.1. *Uses of Equity Swaps*

Equity swaps illustrate the versatility of swap instruments.

6.1.1. Fund Management

There is a huge industry of fund management where the fund manager tries to track some equity *index*. One way to do this is by buying the underlying portfolio of stocks that replicates the index and constantly readjusting it as the market moves, or as new funds are received, or paid by the fund. This is a fairly complex operation. Of course, one can use the S&P 500 futures to accomplish this. But futures contracts need to be rolled over and they require mark-to-market adjustments. Using equity swaps is a cost-effective alternative.

The fund manager could enter into an equity swap using the S&P 500 in which the fund will pay, quarterly, a Libor-related rate *and* a (positive or negative) spread and receive the return on the S&P 500 index for a period of n years.[20]

The example below is similar to the one seen earlier in this chapter. Investors were looking for cost-effective ways to diversify their portfolios.

EXAMPLE:

In one equity swap, the holder of the instrument pays the total return on the S&P 500 and receives the return on the FTSE 100. Its advantage for the investor is the fact that, as a swap, it does not involve paying any up-front premium.

The same position cannot be replicated by selling S&P stocks and buying FTSE 100 stocks.

The second paragraph emphasizes one convenience of the equity swap. Because it is based directly on an index, equity swap payoffs do not have any "tracking error." On the other hand, the attempt to replicate an index using underlying stocks is bound to contain some replication error.

6.1.2. Tax Advantages

Equity swaps are not only "cheaper" and more efficient ways of taking a position on indices, but may have some *tax* and *ownership* advantages as well. For example, if an investor wants to sell a stock that has appreciated significantly, then doing this through an outright sale will be subject to capital gains taxes. Instead, the investor can keep the stock, but, get into an equity swap where he or she pays the capital gains (losses) and dividends acquired from the stock,

[20] The return on the S&P 500 index will be made of capital gains (losses) as well as dividends.

and receives some Libor-related return and a spread. This is equivalent to selling the stock and placing the received funds in an interbank deposit.

6.1.3. Regulations

Finally, equity swaps help in executing some strategies that otherwise may not be possible to implement due to regulatory considerations. In the following example, with the use of equity swaps investment in an emerging market becomes feasible.

EXAMPLE:

Since the Kospi 200 futures were introduced foreign securities houses and investors have been frustrated by the foreign investment limits placed on the instrument. They can only execute trades if they secure an allotment of foreign open interest first, and any allotments secured are lost when the contract expires. Positions cannot be rolled over. Foreigners can only hold 15% of the three-month daily average of open interest, while individual investors with "Korean Investor IDs" are limited to 3%. Recognizing the bottleneck, Korean securities houses such as Hyundai Securities have responded by offering foreign participants equity swaps which are not limited by the restrictions.

The structure is quite simple. A master swap agreement is established between the foreign client and an off-shore subsidiary or a special purpose vehicle of the Korean securities company. Under the master agreement, foreigners execute equity swaps with the off-shore entity which replicate the futures contract. Because the swap transactions involve two non-resident parties and are booked overseas, the foreign investment limits cannot be applied.

The Korean securities houses hedge the swaps in the futures market and book the trades in their proprietary book. Obviously, as a resident entity, the foreign investment limits are not applied to the hedging trade. Once the master swap agreement is established, the foreign client can contact the Korean securities company directly in Seoul, execute any number of trades and have them booked and compiled against the master swap agreement. (IFR, January 27, 1996)

The reading shows how restrictions on (1) ownership, (2) trading, and (3) rolling positions over, can be handled using an equity swap. The reading also displays the structure of the equity swap that is put in place and some technical details associated with it.

6.1.4. Creating Synthetic Positions

The following example is a good illustration of how equity swaps can be used to create synthetic positions.

EXAMPLE:

Equity derivative bankers have devised equity-swap trades to (handle) the regulations that prevent them from shorting shares in Taiwan, South Korea, and possibly Malaysia. The technique is not new, but has re-emerged as convertible bond (CB) issuance has picked up in the region, and especially in these three countries.

Bankers have been selling equity swaps to convertible bond arbitrageurs, who need to short the underlying shares but have been prohibited from doing so by local market regulations.

> *It is more common for a convertible bond trader to take a short equity-swap position with a natural holder of the stock. The stockholder will swap his long stock position for a long equity-swap position. This provides the CB trader with more flexibility to trade the physical shares. When the swap matures—usually one year later—the shares are returned to the institution and the swap is settled for cash. (IFR, December 5, 2001)*

In this example a convertible bond (CB) trader needs to short a security by an amount that changes continuously.[21] A convenient way to handle such operations is for the CB trader to write an equity swap that pays the equity returns to an investor, and gets the investors' physical shares to do the hedging.

6.1.5. Stripping Credit Risk

Suppose we would like to strip the credit risk implicit in a defaultable coupon bond. Note that the main problem is that the bond yield will depend on *two* risks. First is the credit spread and second is the interest rate risk. An asset swap, where we pay a fixed swap rate and receive Libor, will then eliminate the interest rate risk in the bond. The result is called asset swap spread.

6.2. Using Interest Rate and Other Swaps

Interest rate swaps play a much more fundamental role than equity swaps. In fact, all swaps can be used in *balance sheet management*. Balance sheets contain several cash flows; using the swap, one can switch these cash flow characteristics. Swaps are used in *hedging*. They have *zero* value at time of initiation and hence don't require any funding. A market practitioner can easily cover his positions in equity, commodities, and fixed income by quickly arranging proper swaps and then unwind these positions when there is no need for the hedge.

Finally, swaps are also *trading* instruments. In fact, one can construct *spread trades* most conveniently by using various types of swaps. Some possible spread trades are given by the following:

- Pay n-period fixed rate s_{t_0} and receive floating Libor with notional amount N.
- Pay L_{t_i} and receive r_{t_i} both floating rates, in the same currency. This is a basis swap.
- Pay and receive two floating rates in different currencies. This will be a currency swap.

As these examples show, swaps can pretty much turn every interesting instrument into some sort of "spread product." This will reduce the underlying credit risks, make the value of the swap zero at initiation, and, if properly designed, make the position relatively easy to value.

6.3. Two Uses of Interest Rate Swaps

We now consider two examples of the use of interest rate swaps.

6.3.1. Changing Portfolio Duration

Duration is the "average" maturity of a fixed-income portfolio. It turns out that in general the largest fixed-income liabilities are managed by governments, due to the existence of

[21] This is required for the hedging of the implicit option, as will be seen in later chapters.

government debt. Depending on market conditions, governments may want to adjust the average maturity of their debt. Swaps can be very useful here. The following example illustrates this point

EXAMPLE:

France and Germany are preparing to join Italy in using interest rate swaps to manage their debt. Swaps can be used to adjust debt duration and reduce interest rate costs, but government trading of over-the-counter derivatives could distort spreads and tempt banks to front-run sovereign positions. The United States and UK say they have no plans to use swaps to manage domestic debt.

As much as E150 bn of swap use by France is possible over the next couple of years, though the actual figure could be much less, according to an official at the French debt management agency. That is the amount that would be needed if we were to rely on only swaps to bring about "a [significant] shortening of the average duration of our debt," an official said. France has E644.8 bn of debt outstanding, with an average maturity of six years and 73 days.

The official said decisions would be made in September about how to handle actual swap transactions. "If E150 bn was suddenly spread in the market, it could produce an awful mess," he said. (IFR, Issue 1392)

Using swap instruments, similar adjustments to the duration of liabilities can routinely be made by corporations as well.

6.3.2. Technical Uses

Swaps have technical uses. The following example shows that they can be utilized in designing new bond futures contracts where the delivery is tied not to bonds, but to swaps.

EXAMPLE:

LIFFE is to launch its swap-based Libor Financed Bond on October 18. Both contracts are designed to avoid the severe squeeze that has afflicted the Deutsche Terminboerse Bund future in recent weeks.

LIFFE's new contract differs from the traditional bond future in that it is swap-referenced rather than bond-referenced. Instead of being settled by delivery of cash bonds chosen from a delivery basket, the Libor Financed Bond is linked to the International Swap and Derivatives Association benchmark swap rate. At expiry, the contract is cash-settled with reference to this swap rate.

Being cash-settled, the Liffe contract avoids the possibility of a short squeeze—where the price of the cheapest to deliver bond is driven up as the settlement day approaches. And being referenced to a swap curve rather than a bond basket, the contract eliminates any convexity and duration risk. The Libor Financed Bond replicates the convexity of a comparable swap position and therefore reduces the basis risk resulting from hedging with cash bonds or bond futures.

An exchange-based contender for benchmark status, the DTB Bund, has drawn fire in recent weeks following a short squeeze in the September expiry. In the week before, the gross basis between the cheapest to deliver Bund and the Bund future was driven down to -3.5.

> *The squeeze had been driven by a flight to quality on the back of the emerging market crisis. Open interest in the Bund future is above 600,000 contracts or DM15 bn. In contrast, the total deliverable basket for German government bonds is roughly DM74 bn and the cheapest to deliver account to DM30 bn.*
>
> *Officials from the DTB have always contended that there will be no lack of deliverable Bunds. They claim actual delivery has only been made in about 4% of open positions in the past. (IFR Issue 1327)*

In fact, several new cash-settled futures contracts were recently introduced by LIFFE and CME on swaps. Swaps are used as the underlying instrument. Without the existence of liquid swap markets, a swap futures contract would have no such reference point, and would have to be referenced to a bond basket.

7. Mechanics of Swapping New Issues

The swap engineering introduced in this chapter has ignored several minor technical points that need to be taken into account in practical applications. Most of these are minor, and are due to differences in market conventions in bonds, money markets, and swap markets. In this section, we provide a discussion of some of these technical issues concerning interest rate swaps. In other swap markets, such as in commodity swaps, further technicalities may need to be taken into account. A more or less comprehensive list is as follows:

1. Real-world applications of swaps deal with *new* bond issues, and new bond issues imply fees, commissions, and other expenses that have to be taken into account in calculating the true cost of the funds. This leads to the notion of *all-in-cost*, which is different than the "interest rate" that will be paid by the issuer.
   We will show in detail how all-in-cost is calculated, and how it is handled in swapping new issues.
2. Interest rate swaps deal with fixed and floating rates simultaneously. The corresponding Libor is often taken as the floating rate, while the swap rate, or the relevant swap spread is taken as the fixed rate. Another real world complication appears at this point.
   Conventions for quoting money market rates, bond rates, and swap rates usually differ. This requires converting rates defined in one basis, into another.
   In particular, money market rates such as Libor are quoted on an $ACT/360$ basis while *some* bonds are quoted on an annual or semiannual 30/360 basis. In swap engineering, these cash flows are exchanged at regular times, and hence appropriate adjustments need to be made.
3. In this chapter we mostly ignored credit risk. This greatly simplified the exposition because swap rates and corporate rates of similar maturities became equal. In financial markets, they usually are not. Issuers have different credit ratings and bonds sold by them carry credit spreads that are different from the swap spread. This gives rise to new complications in matching cash flows of coupon bonds and interest rate swaps. We need to look at some examples to this as well.
4. Finally, the mechanics of how new issues are swapped into fixed or floating rates and how this may lead to *sub-Libor* financing is an interesting topic by itself.

The discussion will be conducted with a real-life, *new issue*, explained next. First we report the "market reaction" to the bond, and second we have the details of the new issue (see Table 5-1).

TABLE 5-1. Details of the New Issue

Shinhan Bank	
Amount	USD200m
Maturity	3 years (due July 2009)
Coupon	4%
Reoffer price	99.659
Spread at reoffer	168.8 bp over the two-year U.S. Treasury
Launch date	July 23
Payment	July 29
Fees	20 bp
Listing	London
Governing law	London
Negative pledge	Yes
Cross-default	Yes
Sales restrictions	U.S., UK, South Korea
Joint lead managers	ABN AMRO, BNP Paribas, UBS Warburg

Source: IFR issue 1444

EXAMPLE:

South Korea's Shinhan Bank, rated Baa1/BBB by Moody's and S&P, priced its US$200m three-year bond early last week (. . .). The deal came with a 4% coupon and offered a spread of 168.8 bp over the two-year U.S. Treasury, equivalent to 63 bp over Libor.

This was some 6 bp wide of the Korea Development Bank (KDB) curve, although it was the borrower's intention to price flat to it. Despite failing to reach this target, the borrower still managed to secure a coupon that is the lowest on an Asian bond deal since the regional crisis, thanks to falling U.S. Treasury yields which have shrunk on a renewed flight to safe haven assets. (IFR, issue 1444)

Consider now the basic steps of swapping this new issue into floating USD funds.[22] The issuer has to enter into a 3-year interest rate swap agreement. How should this be done, and what are the relevant parameters? Suppose at the time of the issue the market makers were quoting the swap spreads shown in Table 5-2. First we consider the calculation of all-in-cost for the preceding deal.[23]

7.1. *All-in-Cost*

The information given in the details of the new issue implies that the coupon is 4%. But, this is not the true costs of funds from the point of view of the issuer. There are at least three additional factors that need to be taken into account. (1) The reoffer price is not 100, but 99.659. This means that for each bond, the issuer will receive less cash than the par. (2) Fees have to be paid. (3) Although not mentioned in the information in Table 5-1, the issuer has legal and documentation expenses. We assume that these were USD75,000.

To calculate the fixed all-in-cost (30/360 basis), we have to calculate the *proceeds* first. Proceeds is the net cash received by the issuer after the sale of the bonds. In our case, using the

[22] The actual process may differ slightly from our simplified discussion here.

[23] Liquid swaps are against 3 or 6 month Libor. Here we use 12 month Libor for notational simplicity.

TABLE 5-2. USD Swap index versus 12M Libor, Semi, 30/360F

Maturity	Bid spread	Ask spread	The bid-ask swap rate
2-Year	42	46	2.706–2.750
3-Year	65	69	3.341–3.384
4-Year	70	74	3.796–3.838
5-Year	65	69	4.147–4.187
7-Year	75	79	4.653–4.694
10-Year	61	65	5.115–5.159
12-Year	82	86	5.325–5.369
15-Year	104	108	5.545–5.589
20-Year	126	130	5.765–5.809
30-Year	50	54	5.834–5.885

terminology of Table 5-1,

$$\text{Proceeds} = \text{Amount} \times \left(\frac{\text{Price}}{100} - \text{fees}\right) - \text{Expenses} \tag{55}$$

Plugging in the relevant amounts,

$$\text{Proceeds} = 200{,}000{,}000(.99659 - .0020) - 75{,}000 \tag{56}$$

$$= 198{,}843{,}000 \tag{57}$$

Next, we see that the bond will make three coupon payments of 8 million each. Finally, the principal is returned in 3 years. The cash flows associated with this issue are summarized in Figure 5-17. What is the internal rate of return of this cash flow? This is given by the formula

$$198{,}843{,}000 = \frac{8{,}000{,}000}{(1+y)} + \frac{8{,}000{,}000}{(1+y)^2} + \frac{8{,}000{,}000 + 200{,}000{,}000}{(1+y)^3} \tag{58}$$

The y that solves this equation is the internal rate of return. It can be interpreted as the true cost of the deal, and it is the fixed all-in-cost under the (30/360) day-count basis. The calculation gives

$$y = 0.04209 \tag{59}$$

This is the fixed all-in-cost.

The next step is to swap this issue into floating and obtain the floating all-in-cost. Suppose we have the same notional amount of \$200 million and consider a fixed to floating 3-year interest rate swap. Table 5-2 gives the 3-year *receiver* swap rate as 3.341%. This is, by definition, a 30/360, semiannual rate.

This requires converting the semiannual swap rate into an annual 30/360 rate, denoted by r. This is done as follows:

$$(1+r) = \left(1 + .03341\frac{1}{2}\right)^2 \tag{60}$$

which gives

$$r = 3.369\% \tag{61}$$

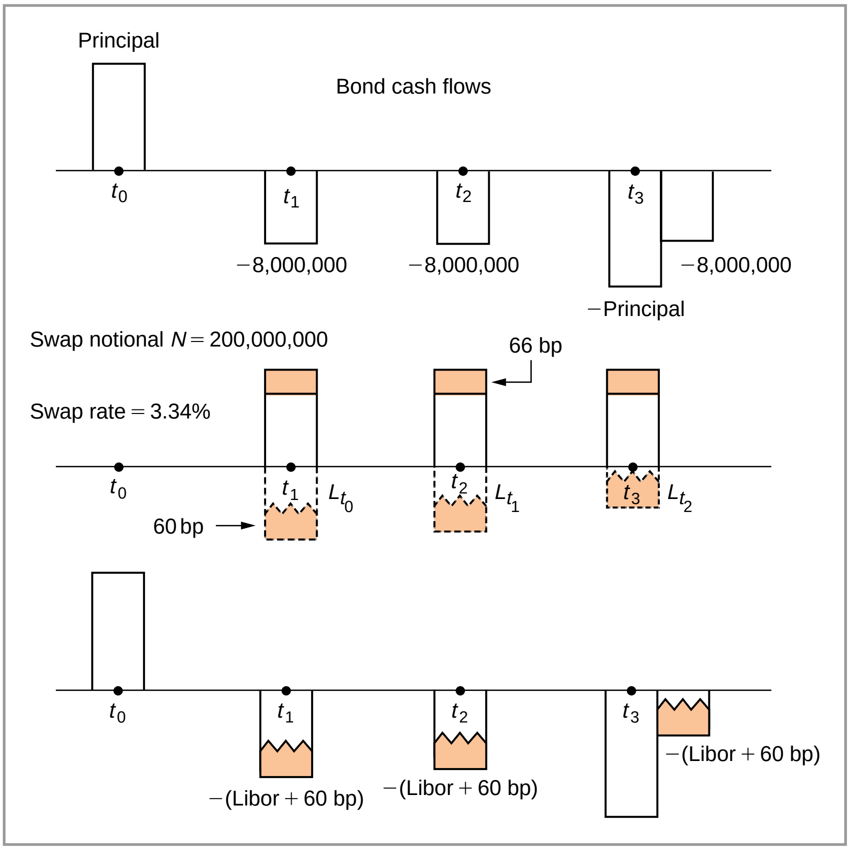


FIGURE 5-17

With a \$200 million notional this is translated into three fixed receipts of

$$200{,}000{,}000(.03369) = 6{,}738{,}000 \tag{62}$$

each. The cash flows are shown in Figure 5-17.

Clearly, the fixed swap receipts are not equal to the fixed annual coupon payments, which are \$8 million each. Apparently, the issuer pays a higher rate than the swap rate due to higher credit risk. To make these two equal, we need to increase the fixed receipts by

$$8{,}000{,}000 - 6{,}738{,}000 = 1{,}262{,}000 \tag{63}$$

This can be accomplished by increasing both the floating rate paid and the fixed rate received by equivalent amounts. This can be accomplished if the issuer accepts paying Libor *plus* a spread equivalent to the 66 bp. Yet, here the 66 bp is p.a. 30/360, whereas the Libor concention is p.a. ACT/360. So the basis point difference of 66 bp may need to be adjusted further.[24] The final figure will be the floating all-in-cost and will be around 60 bp.

[24] Our calculations provided a slightly different number than the 63 basis points mentioned in the market reaction mainly because we used a swap against 12-month Libor for simplicity.

7.2. *Another Example*

Suppose there is an A-rated British entity that would like to borrow 100m sterling (GBP) for a period of 3 years. The entity has no preference toward either floating or fixed-rate funding, and intends to issue in Euromarkets. Market research indicates that if the entity went ahead with its plans, it could obtain fixed-rate funds at 6.5% annually.[25] But the bank recommends the following approach.

It appears that there are nice *opportunities* in USD-GBP currency swaps, and it makes more sense to issue a floating rate Eurobond in the USD sector with fixed coupons. The swap market quotes funding at Libor + 95 bp in GBP against USD rates for this entity. Then the proceeds can be swapped into sterling for a lower all-in-cost. How would this operation work? And what are the risks?

We begin with the data concerning the new issue. The parameters of the newly issued bond are in Table 5-3.[26] Now, the issuer would like to swap these proceeds to floating rate GBP funds. In doing this, the issuer faces the market conditions shown in Table 5-4. We first work out the original and the swapped cash flows and then calculate the all-in-cost, which is the real cost of funds to the issuer after the proceeds are swapped into GBP.

The first step is to obtain the amount of cash the issuer will receive at time t_0 and then determine how much will be paid out at t_1, t_2. To do this, we again need to calculate the *proceeds* from the issue.

TABLE 5-3. The New Issue

Amount	USD100 million
Maturity	2 years
Coupon	6% p.a.
Issue price	$100\frac{3}{4}$
Options	none
Listing	Luxembourg
Commissions	$1\frac{1}{4}$
Expenses	USD75000
Governing law	English
Negative pledge	Yes
Pari passu	Yes
Cross default	Yes

TABLE 5-4. Swap Market Quotes

Spot exchange rate GBP-USD	$1.6701/1.6708$
GBP 2-year interest rate swap	$5.46/51$
USD-GBP currency swap	$+4/-1$

[25] With a day-count basis of 30/360.

[26] Some definitions: *Negative pledge* implies that the investors will not be put in a worse position at a later date by the issuer's decision to improve the risks of other bonds. *Pari passu* means that no investor who invested in these bonds will have an advantageous position. *Cross default* means the bond will be considered in default even if there is time to maturity and if the issuer defaults on another bond.

- The issue price is 100.75 and the commissions are 1.25%. This means that the amount received by the issuer before expenses is

$$\frac{(100.75 - 1.25)}{100} 100{,}000{,}000 = 99{,}500{,}000 \tag{64}$$

We see that the issue is sold at a premium which increases the proceeds, but once commissions are deducted, the amount received falls below 100 million. Thus, expenses must be deducted

$$\text{Proceeds} = 99{,}500{,}000 - 75{,}000 = 99{,}425{,}000 \tag{65}$$

Given the proceeds, we can calculate the effective cost of fixed rate USD funds for this issuer. The issuer makes two coupon payments of 6% (out of the 100 million) and then pays back 100 million at maturity. At t_0, the issuer receives only 99,425,000. This cash flow is shown in Figure 5-18. Note that unlike the theoretical examples, the principal paid is not the same as the principal received. This is mainly due to commissions and expenses.

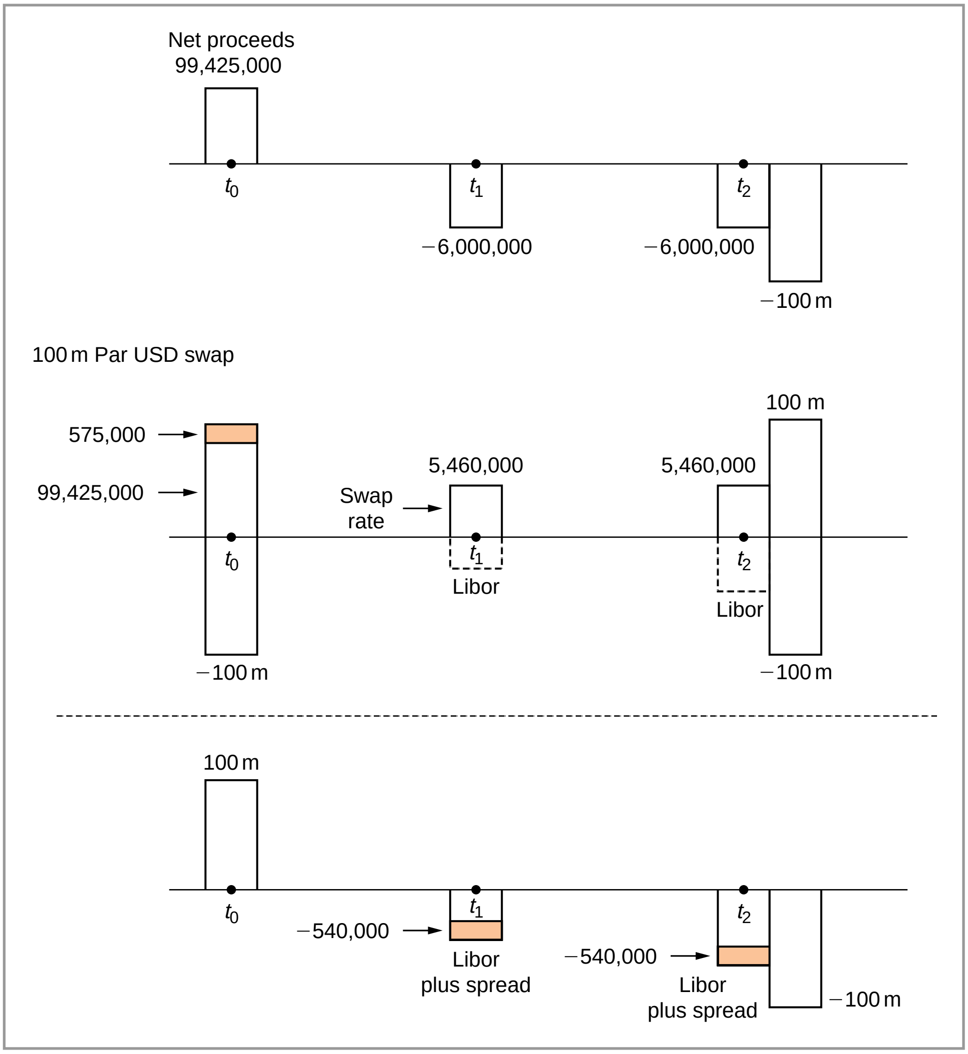


FIGURE 5-18

- From this cash flow we can calculate the internal rate of return y_{t_0} by solving the equation

$$99{,}425{,}000 = \frac{60{,}000}{(1+y_{t_0})} + \frac{60{,}000}{(1+y_{t_0})^2} + \frac{100{,}000{,}000}{(1+y_{t_0})^2} \tag{66}$$

  The solution is

$$y_{t_0} = 6.3150\% \tag{67}$$

  Hence, the true fixed cost of USD funds is greater than 6%.

The issuer will first convert this into floating rate USD funds. For this purpose, the issuer will *sell* a swap. That is to say, the issuer will receive fixed 5.46% and pay floating Libor flat. This is equivalent to paying approximately USD Libor + 54 bp. Finally, the issuer will convert these USD floating rate funds into GBP floating rate funds by paying floating GBP and receiving floating USD.

8. Some Conventions

If you have a coupon bond and the payment date falls on a nonworking day, then the payment will be made on the first following working day. But the amount does not change. In swaps, this convention is slightly different. The payment is again delayed to the next working day.[27] But the payment amount will be adjusted according to the actual number of days. This means that the payment dates and the amounts may not coincide exactly in case swaps are used as hedges for fixed-income portfolios.

8.1. *Quotes*

Suppose we see quotes on interest rate swaps or some other liquid swap. Does this mean we can deal on them? Not necessarily. Observed swap rates may be available as such only to a bank's best customers; others may have to pay more. In practice, the bid-ask spreads on liquid instruments are very tight, and a few large institutions dominate the market.

9. Currency Swaps versus FX Swaps

We will now compare currency swaps with FX-swaps introduced in Chapter 3. A currency swap has the following characteristics:

1. Two principals in different currencies and of equal value are exchanged at the start date t_0.
2. At settlement dates, interest will be paid and received in different currencies, and according to the agreed interest rates.
3. At the end date, the principals are re-exchanged at the *same* exchange rate.

A simple example is the following. 100,000,000 euros are received and against these 100,000,000 e_{t_0} dollars are paid, where the e_{t_0} is the "current" EUR/USD exchange rate. Then,

[27] If this next day is in the following calendar month, then the payment is made during the *previous* working day.

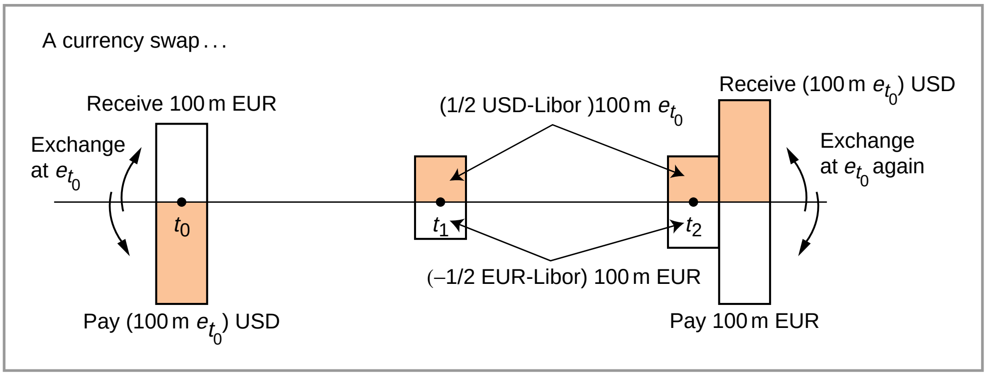


FIGURE 5-19

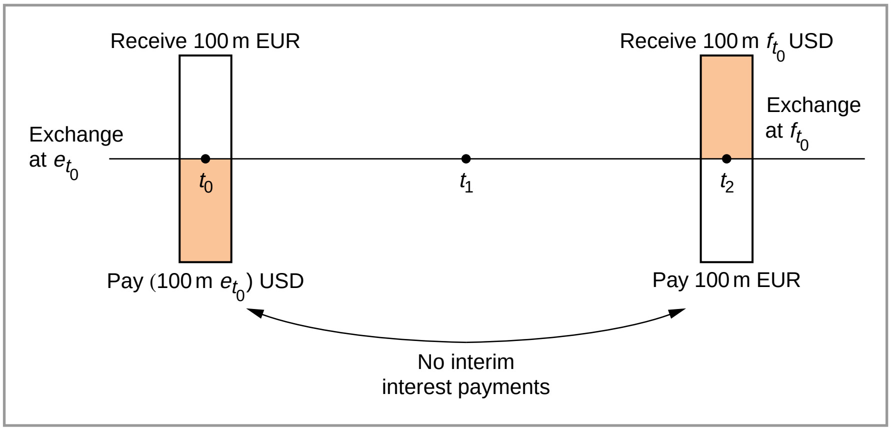


FIGURE 5-20

6-month Libor-based interest payments are exchanged twice. Finally, the principal amounts are exchanged at the end date *at the same* exchange rate e_{t_0}, even though the actual exchange rate e_{t_2} at time t_2 may indeed be different than e_{t_0}. See Figure 5-19.

The FX-swap for the same period is in Figure 5-20. Here, we have no interim interest payments, but instead the principals are reexchanged at a *different* exchange rate equal to

$$f_{t_0} = e_{t_0} \frac{1 + L_{t_0}^{\mathrm{USA}} \delta}{1 + L_{t_0}^{\mathrm{EUR}} \delta} \tag{68}$$

Why this difference? Why would the same exchange rate be used to exchange the principals at start and end dates of a currency swap while different exchange rates are used for an FX-swap?

We can look at this question from the following angle. The two parties are exchanging currencies for a period of 1 year. At the end of the year they are getting back their original currency. But during the year, the interest rates in the two currencies would normally be different. This difference is explicitly paid out in the case of currency swaps during the life of the swap as interim interest payments. As a result, the counterparties are ready to receive the exact original amounts back. The interim interest payments would compensate them for any interest rate differentials.

In the case of FX-swaps, there are no interim interest payments. Hence, the compensation must take place at the end date. Thus, the interest payments are bundled together with the exchange of principals at the end date.

9.1. Another Difference

Looked at from a financial engineering perspective, the currency swap is like an exchange of two FRNs with different currencies and no credit risk. The FX-swap, on the other hand, is like an exchange of two zero-coupon bonds in different currencies.

Because the Libor rates at time t_1 are unknown as of time t_0, the currency swap is subject to slightly different risks than FX-swaps of the same maturity.

10. Additional Terminology

We would like to introduce some additional terms and instruments before moving on.

A *par swap* is the formal name of the interest rate swaps that we have been using in this chapter. It is basically a swap structure calculated over an initial and final (nominal) exchange of a principal equal to 100. This way, there will be no additional cash payments at the time of initiation.

An *accrual swap* is an interest rate swap in which one party pays a standard floating reference rate, such as Libor, and receives Libor plus a spread. But the interest payments to the counterparty will accrue only for days on which Libor stays within preset upper and lower boundaries.

A *commodity-linked interest rate swap* is a hybrid swap in which Libor is exchanged for a fixed rate, linked to a commodity price. A buyer of crude oil may wish to tie costs to the cost of debt. The buyer could elect to receive Libor and pay a crude oil-linked rate such that, as the price of crude oil rises, the fixed rate the buyer pays declines.

A *crack spread swap* is a swap used by oil refiners. They pay the floating price of the refined product and receive the floating price of crude oil plus a fixed margin, the crack spread. This way, refiners can hedge a narrowing of the spread between crude oil prices and the price of their refined products.

An *extendible swap* is a swap in which one party has the right to extend a swap beyond its original term.

A *power Libor swap* is a swap that pays Libor squared or cubed (and so on), less a fixed amount/rate, in exchange for a floating rate.

10.1. Two Useful Concepts

There are some standard bond market terms that are often used in swap markets. We briefly review some of them here. The *present value of a basis point* or $PV01$ is the present value of an annuity of 0.0001 paid periodically at times t_i, calculated using the proper Libor rates, or the corresponding forward rates:

$$PV01 = \sum_{i=1}^{n} \frac{(.01)\delta}{\prod_{j=1}^{i}(1 + L_{t_j}\delta)} \tag{69}$$

In order to get the sensitivity to one basis point, the figure obtained from this formula needs to be divided by 100.

The $DV01$ is the *dollar value of a 1 bp change* in yield, y_{t_0}. This is expressed as

$$DV01 = m(\text{Bond price})(.01) \tag{70}$$

where m is the modified duration defined as the derivative of a bond's price with respect to yield, divided by the bond price. The concepts, $PV01$ and $DV01$ are routinely used by market practitioners for the pricing of swap-related instruments and other fixed-income products.[28]

11. Conclusions

Why buy and sell securities when you can swap the corresponding returns and achieve the same objective efficiently, and at minimal cost?

In fact, selling or buying a security may not be practical in many cases. First, these operations generate cash which needs to be taken care of. Second, the security may not be very liquid and selling it may not be easy. Third, once a security is sold, search costs arise when, for some reason, we need it back. Can we find it? For how much? What are the commissions? Swapping the corresponding returns may cost less.

Due to their eliminating the need to use cash in buying and selling transactions, combining these two operations into one, and eliminating potential credit risks, swaps have become a major tool for financial engineers.

Suggested Reading

*Swaps are vanilla products, and there are several recommended books that deal with them. This chapter has provided a nontechnical introduction to swaps, hence we will list references at the same level. For a good introduction to swap markets, we recommend **McDougall** (1999). **Flavell** (2002) and **Das** (1994) give details of swaps and discuss many examples. **Cloyle** (2000) provides an introduction to the basics of currency swaps.*

[28] In the preceding formulas, the "bond price" always refers to *dirty price* of a bond. This price equals the true market value, which is denoted by "clean price" plus any accrued interest.

Exercises

1. You have a 4-year coupon bond that pays semiannual interest. The coupon rate is 8% and the par value is 100.

    (a) Can you construct a synthetic equivalent of this bond? Be explicit and show your cash flows.
    (b) Price this coupon bond assuming the following term structure: $B_1 = .90/.91, B_2 = .87/.88, B_3 = .82/.83, B_4 = .80/81$
    (c) What is the 1×2 FRA rate?

2. Read the following episode carefully.

    Italian Asset Swap Volumes Soar on Buyback Plans

    Volumes in the basis-swap spread market doubled last week as traders entered swaps in response to the Italian treasury's announcement that it "does not rule out buybacks." Traders said the increase in volume was exceptional given that so many investors are on holiday at this time of year.

    Traders and investors were entering trades designed to profit if the treasury initiates a buyback program and the bonds increase in value as they become scarcer and outperform the swaps curve. A trader said in a typical trade the investor owns the 30-year Italian government bond and enters a swap in which it pays the 6% coupon and receives 10.5 basis points over six-month Euribor. "Since traders started entering the position last Monday the spread has narrowed to 8 bps over Euribor," he added. The trader thinks the spread could narrow to 6.5 bps over Euribor within the next month if conditions in the equity and emerging markets improve. A trader at a major European bank predicts this could go to Euribor flat over the next six months. The typical notional size of the trades is EUR50 million (USD43.65 million) and the maturity is 30 years. (IFR, Issue 1217)

    (a) Suppose there is an Italian swap curve along with a yield curve obtained from Italian government bonds (sovereign curve). Suppose this latter is upward sloping. Discuss how the two curves might shift relative to each other if the Italian government buys back some bonds.
    (b) Is it important which bonds are bought back? Discuss.
    (c) Show the cash flows of a 5-year Italian government coupon bonds (paying 6%) and the cash flows of a fixed-payer interest-rate swap.
    (d) What is the reason behind the existence of the 10.15 bp spread?
    (e) What happens to this spread when government buys back bonds? Show your conclusions using cash flow diagrams.

3. You are a swap dealer and you have the following deals on your book:

    Long

    - 2-year receiver vanilla interest rate swap, at 6.75% p.a. 30/360. USD $N = 50$ million.
    - 3-year receiver vanilla interest rate swap, at 7.00% p.a. 30/360. USD $N = 10$ million.

Short

- 5-year receiver vanilla interest rate swap, at 7.55% p.a. 30/360. USD $N = 10$ million.

(a) Show the cash flows of each swap.
(b) What is your net position in terms of cash flows? Show this on a graph.
(c) Calculate the present values of each swap using the swap curve:

Maturity	Bid-ask
2	6.75–6.80
3	6.88–6.92
4	7.02–7.08
5	7.45–7.50
6	8.00–8.05

(d) What is your net position in terms of present value?
(e) How would you hedge this with a 4-year swap? Which position would you take, and what should the notional amount be?
(f) Where would you go to get this hedge?
(g) Can you suggest another hedge?

4. Suppose at time $t = 0$, we are given prices for four zero-coupon bonds (B_1, B_2, B_3, B_4) that mature at times $t = 1, 2, 3$, and 4. This forms the term structure of interest rates.

   We also have the *one-period* forward rates (f_0, f_1, f_2, f_3), where each f_i is the rate contracted at time $t = 0$ on a loan that begins at time $t = i$ and ends at time $t = i + 1$. In other words, if a borrower borrows N GBP at time i, he or she will pay back $N(1 + f_i)$ GBP at time $t = i + 1$.

   The spot rates are denoted by r_i. By definition we have

$$r = f_0 \tag{71}$$

   The $\{B_i\}$ and all forward loans are default-free, so that there is no credit risk. You are given the following *live* quotes:

$$B_1 = .92/.94, B_2 = .85/.88, B_3 = .82/.85 \tag{72}$$

   and

$$f_0 = 8.10/8.12, f_1 = 9.01/9.03, f_2 = 10.12/10.16, f_3 = 18.04/18.10 \tag{73}$$

   (a) Given the data on forward rates, obtain arbitrage-free prices for the zero-coupon bonds, B_1 and B_2.
   (b) What is the three-period swap rate under these conditions?

5. Going back to Question 4, suppose you are given, in addition, data on FRAs both for USD and for EUR. Also suppose you are looking for arbitrage opportunities. Would these additional data be relevant for you? Discuss briefly.
6. Foreigners buying Australian dollar instruments issued in Australia have to pay withholding taxes on interest earnings. This withholding tax can be exploited in tax-arbitrage portfolios using swaps and bonds. First let us consider an episode from the markets related to this issue.

Under Australia's withholding tax regime, resident issuers have been relegated to second cousin status compared with non resident issuers in both the domestic and international markets. Something has to change.

In the domestic market, bond offerings from resident issuers incur the 10% withholding tax. Domestic offerings from non resident issuers, commonly known as Kangaroo bonds, do not incur withholding tax because the income is sourced from overseas. This raises the spectre of international issuers crowding out local issuers from their own markets.

In the international arena, punitive tax rules restricting coupon washing have reduced foreign investor interest in Commonwealth government securities and semi-government bonds. This has facilitated the growth of global Australian dollar offerings by Triple A rated issuers such as Fannie Mae, which offer foreign investors an attractive tax-free alternative.

The impact of the tax regime is aptly demonstrated in the secondary market. Exchangeable issues in the international markets from both Queensland Treasury Corporation and Treasury Corporation of NSW are presently trading through comparable domestic issues. These exchangeable issues are exempt from withholding tax.

If Australia wishes to develop into an international financial centre, domestic borrowers must have unfettered access to the international capital markets—which means compliance costs and uncertainty over tax treatment must be minimized. Moreover, for the Australian domestic debt markets to continue to develop, the inequitable tax treatment between domestic and foreign issues must be corrected. (IFR, Issue 1206)

We now consider a series of questions dealing with this problem. First, take a 4-year straight coupon bond issued by a local government that pays interest annually. We let the coupon rate be denoted by c%.

Next, consider an Aussie dollar Eurobond issued at the same time by a Spanish company. The Eurobond has a coupon rate d%. The Spanish company will use the funds domestically *in Spain*.

Finally, you know that interest rate swaps or FRAs in Aussie$ are not subject to any tax.

(a) Would a foreign investor have to pay the withholding tax on the Eurobond? Why or why not?
(b) Suppose the Aussie$ IRSs are trading at a swap rate of $d + 10$ bp. Design a 4-year interest rate swap that will benefit from tax arbitrage. Display the relevant cash flows.
(c) If the swap notional is denoted by N, how much would the tax arbitrage yield?
(d) Can you benefit from the same tax-arbitrage using a strip of FRAs in Aussie$?
(e) Which arbitrage portfolio would you prefer, swaps or FRAs? For what reasons?
(f) Where do you think it is more profitable for the Spanish company to issue bonds under these conditions, in Australian domestic markets or in Euromarkets? Explain.

7. Consider a 2-year currency swap between USD and EUR involving floating rates only. The EUR benchmark is selected as 6-month Euro Libor, the dollar benchmark is 6-month BBA Libor. You also have the following information:

Notional amount = USD10,000,000
Exchange rate EUR/USD = 0.84

(a) Show the cash flow diagrams of this currency swap. Make sure to quantify every cash flow exactly (i.e., use a graph as well as the corresponding number).
(b) Show that this currency swap is equivalent to two floating rate loans.
(c) Suppose a company is trying to borrow USD10,000,000 from money markets.

The company has the following information concerning available rates on 6-month loans:

EUR Libor = 5.7%, USD Libor = 6.7%

EUR-USD currency swap spread: 1 year -75, 2 years -90.

Should this company borrow USD directly? Would the company benefit if it borrowed EUR first and then swapped them into USD?